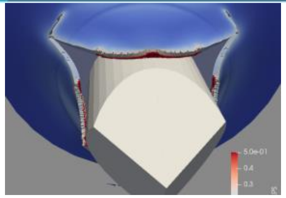
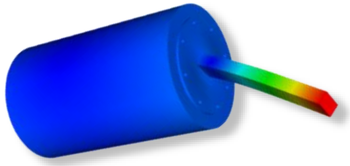
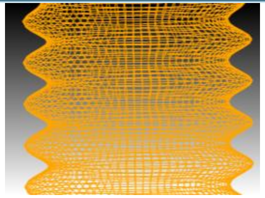
Extracting the Salient Features of a Multi-Harmonic Time Response with Closely Spaced Modes



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August 6th, 2024

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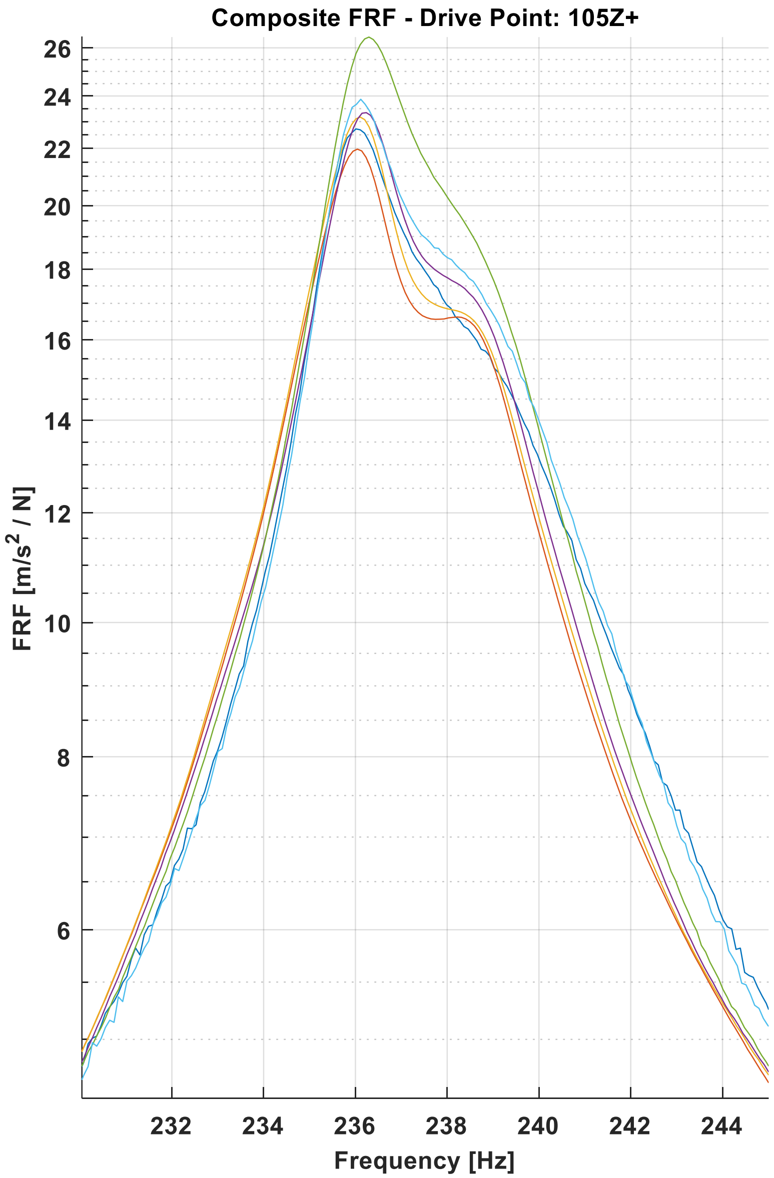
SAND2024-10180PE

Project Overview

- Identify resonant failure modes and allowable energy dissipation
- Nonlinearity: frequency and damping change with amplitude
- Finding the natural frequency and damping of closely spaced nonlinear modes can be difficult with current methods
- Great opportunity to test the limits of current methods. How close is too close? Better methods, can create designs leveraging nonlinearity

Nonlinearity – Leveraged to adapt system response to different environmental resonant conditions

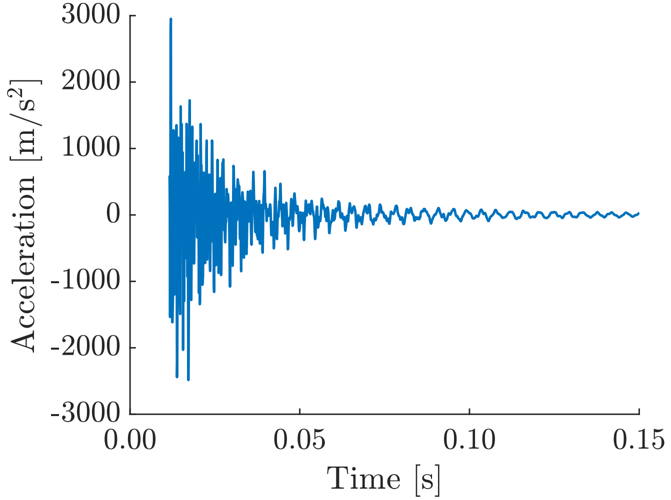
https://www.britannica.com/technology/jet-engine

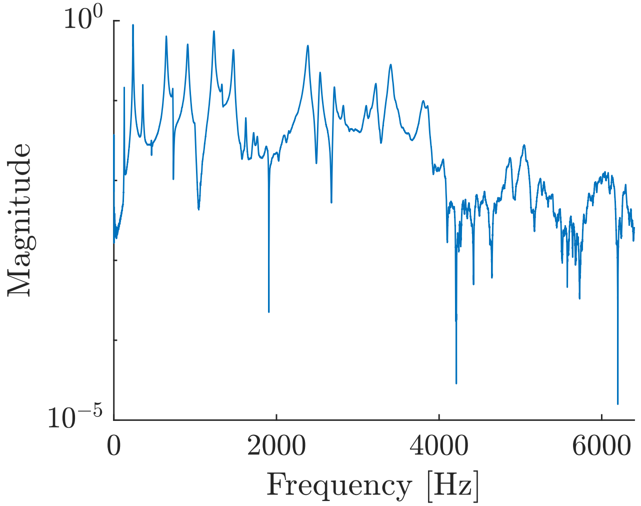


Methods of Analysis

Mono-Harmonic Signals from Multi-Harmonic Responses

$$\ddot{q} = \phi^{-1}\ddot{x}$$



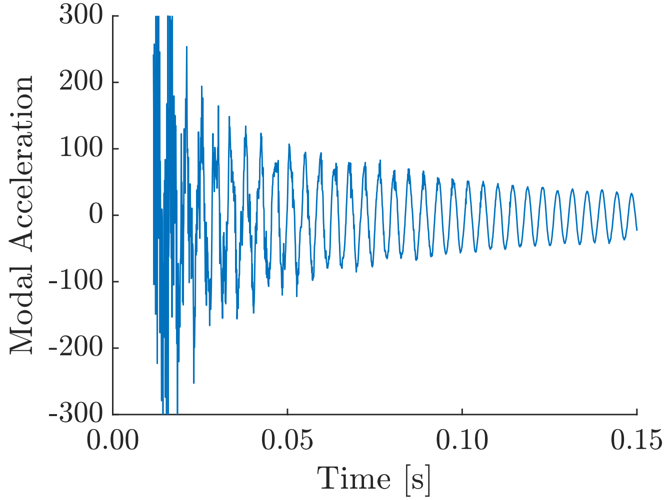


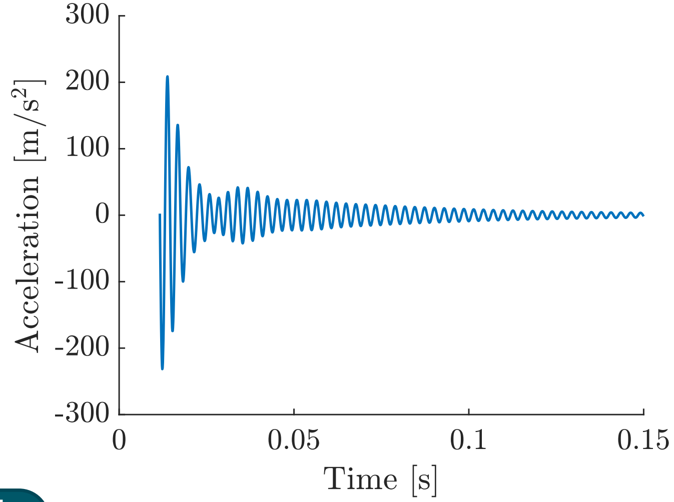
Start with multi-harmonic response

Modal

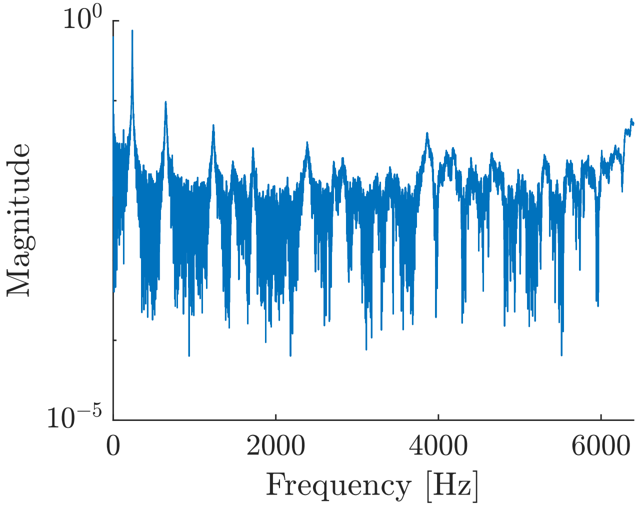
Bandpass

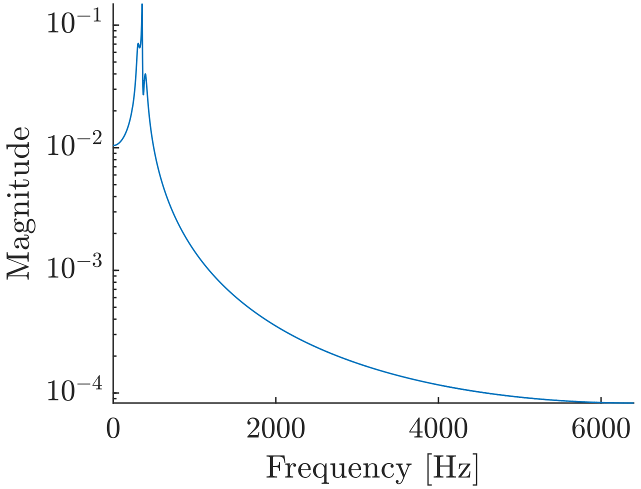
SI units used throughout presentation





Filtered response in time domain

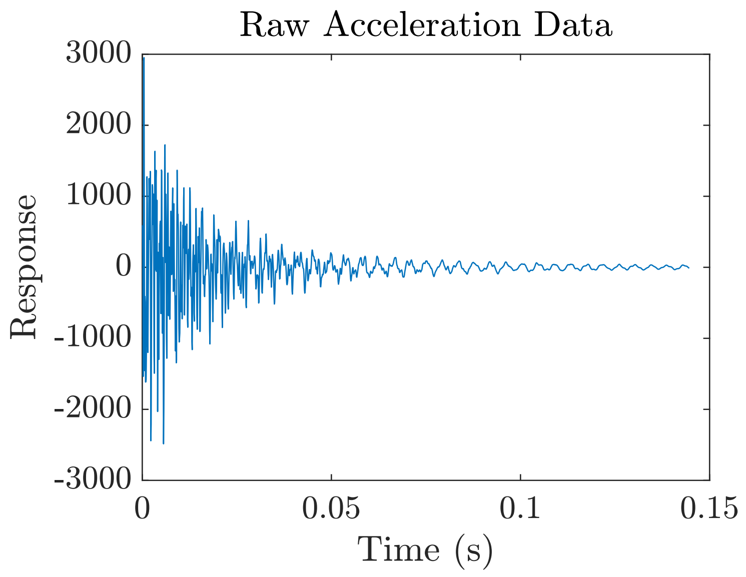


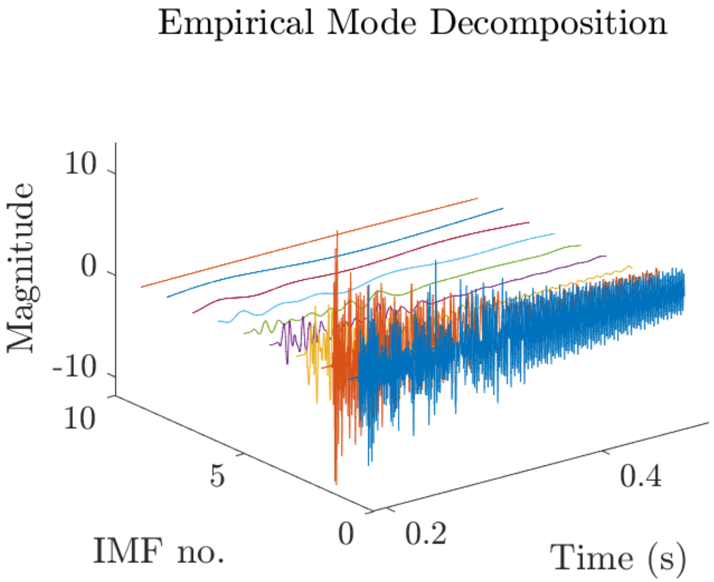


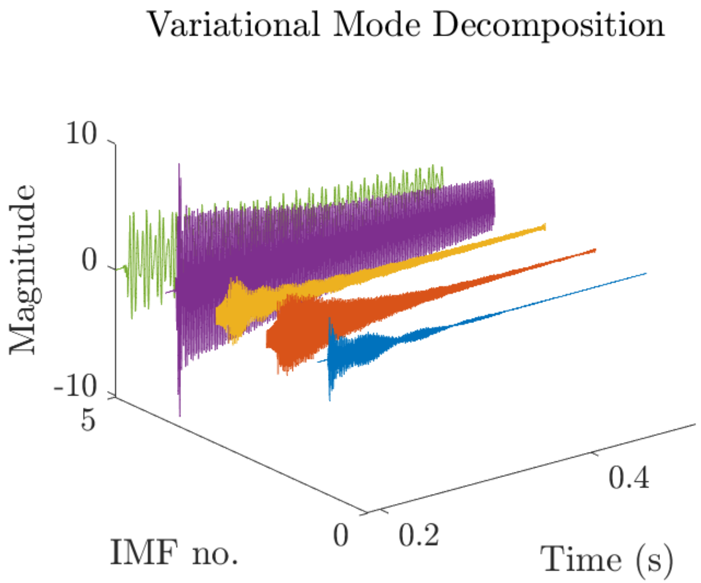
Filtered response in frequency domain

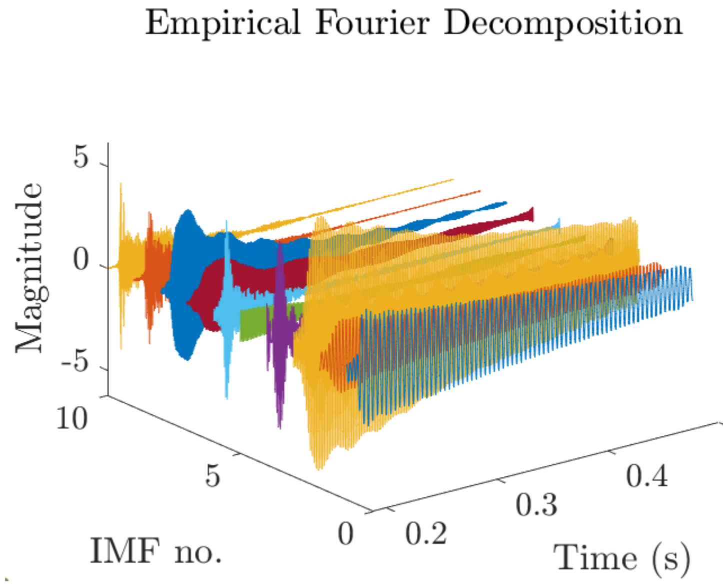
Alternative Modal Decomposition Methods

- Empirical Mode Decomposition (EMD) [1]
  - Sifting local min/max - frequency band range
- Variational Mode Decomposition (VMD) [1]
  - Limited band range
- Empirical Fourier Decomposition (EFD) [2]
  - Fourier peak band range selection









Intrinsic mode functions (IMFs) identified from decompositions – extract range of nonlinear system responses

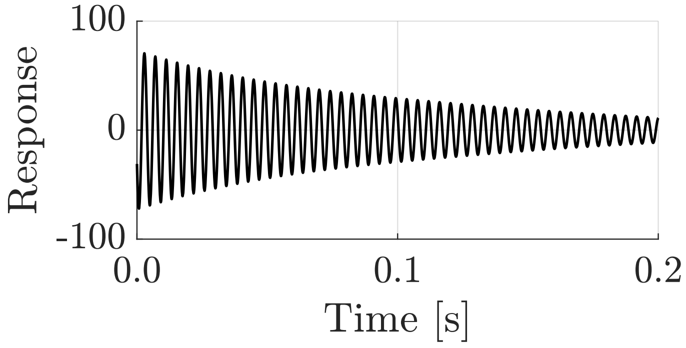
Hilbert Transform: Mono-harmonic

- Begin with real-valued signal, $y(t)$, and view this as the real part of a complex signal [3]

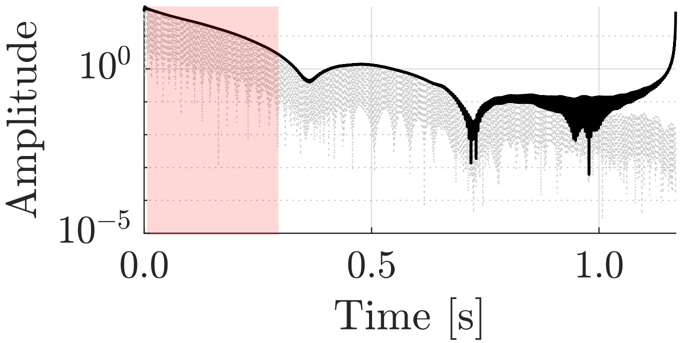
$$Y(t) = y(t) + i\tilde{y}(t)$$

- Take time derivative of amplitude $A(t)$ and instantaneous angular frequency $\omega(t)$ to find natural frequency $f_n(t)$ and damping ratio $\zeta(t)$ [4]

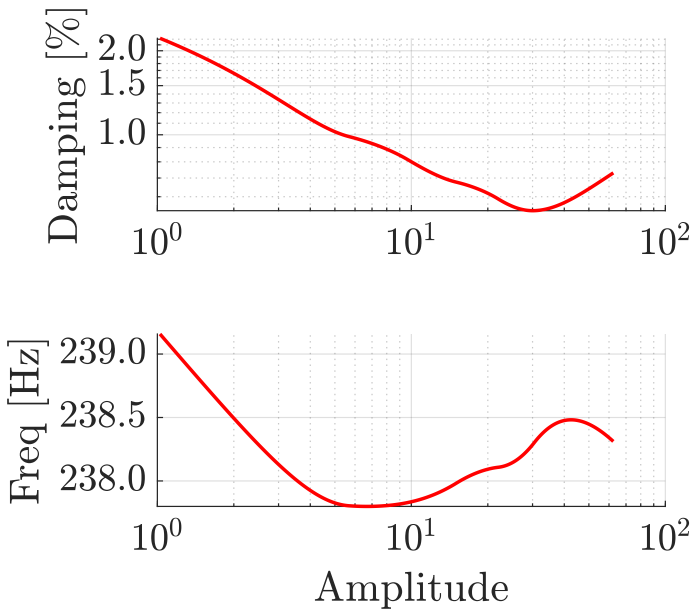
$$f_n(t) = \frac{1}{2\pi}\sqrt{\omega^2 - \frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} + \frac{\dot{A}\dot{\omega}}{A\omega}} \qquad \zeta(t) = \frac{1}{2\pi f_n}\left(-\frac{\dot{A}}{A} - \frac{\dot{\omega}}{2\omega}\right)$$



Mono-harmonic input



Fit amplitude envelope in appropriate region



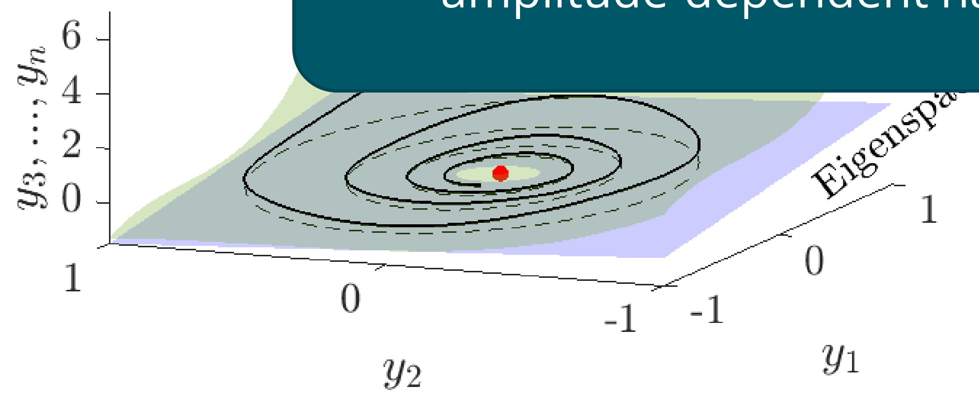
Damping and frequency curves

Spectral Submanifolds (SSMs): Mono-harmonic

- SSMs are nonlinear continuations of linear eigenspaces that capture nonlinear dynamics
- Data-driven MATLAB package **SSMLearn** computes reduced-order models (ROMs) using SSMs [7]
- Damping and natural frequency vs. amplitude can be extracted from ROMs

Physical space ($\mathbf{x}$) → Observable space ($\mathbf{y}$) → Reduced coordinates ($\boldsymbol{\eta}$) → Polar coordinates ($\boldsymbol{\rho}$, $\boldsymbol{\theta}$)

Goal: Evaluate the performance of SSMLearn in quantifying amplitude-dependent natural frequency and damping ratio



Conceptual SSM visualization

form reads

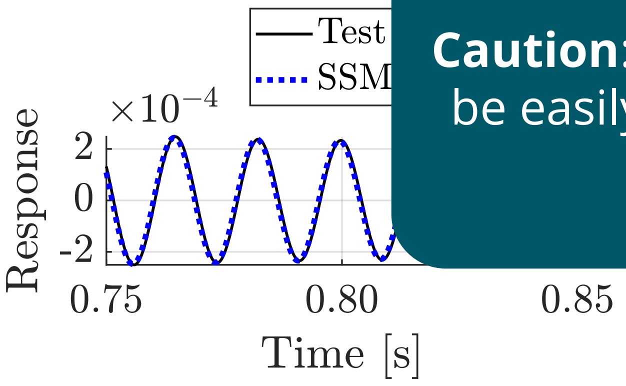
$$\dot{\rho}\rho^{-1} = -1.1906 - 46.3984\rho^2 + 662.7702\rho^4 - 5116.8725\rho^6 + 18523.3219\rho^8 - 24889.5404\rho^{10}$$

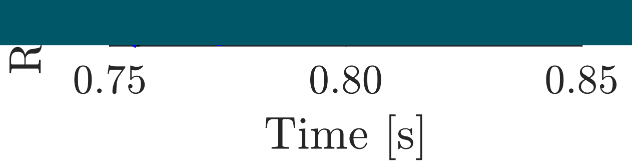
$$\dot{\theta} = +359.4611 - 183.0644\rho^2 + 2728.4845\rho^4 - 22138.0385\rho^6 + 85198.2641\rho^8 - 123421.0798\rho^{10}$$

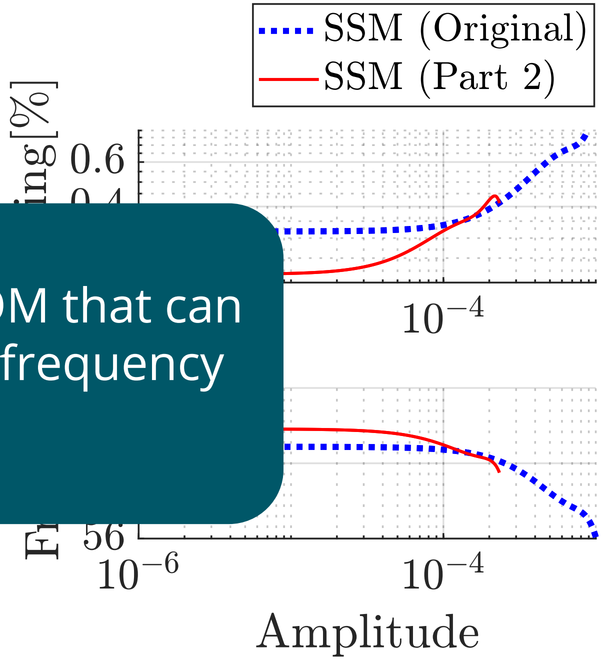
Sample ROM in normal form style

Split SSMLearn (sSSMLearn): Mono-harmonic

- One of the challenges of SSMLearn is that the predicted damping and natural frequency become inaccurate at very low amplitudes
- **Split SSMLearn** remedies this by fitting separate reduced-order models to the low amplitude portions of responses







Caution: Split SSMLearn no longer produces a single ROM that can be easily integrated. It was developed only to generate frequency and damping curves.

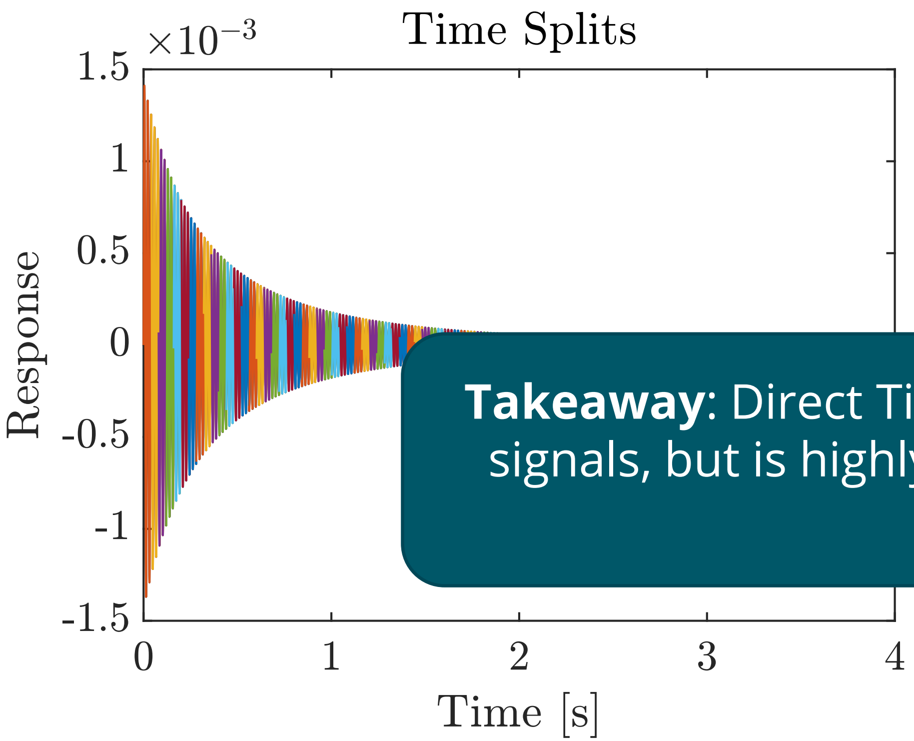
Fit one ROM to the full response

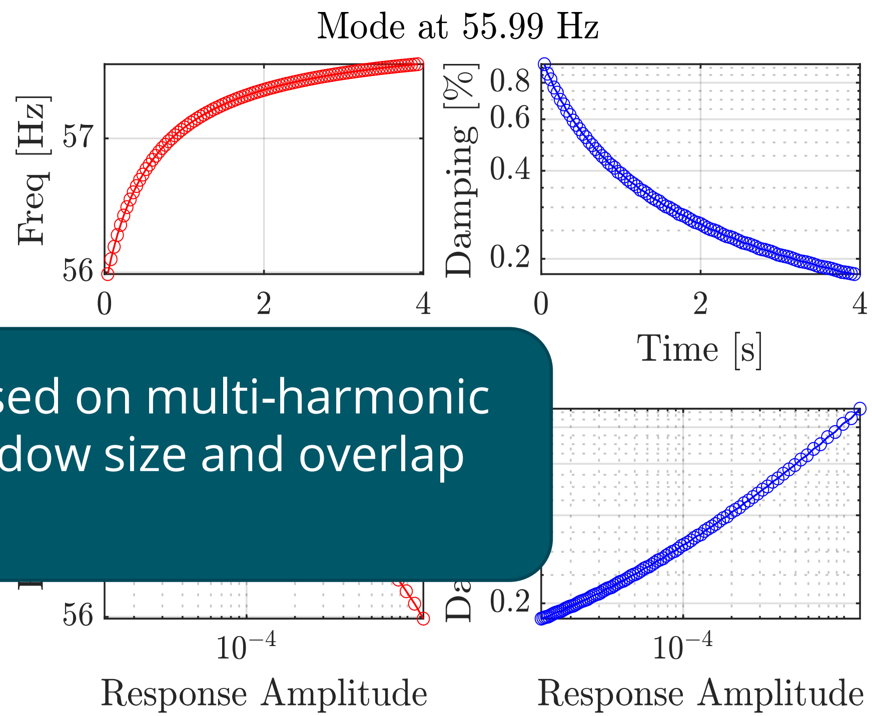
Fit another ROM to the later part of the response

Plot damping and frequency from both ROMs in the same figure

Direct Time Fitting (DTimeFit): Multi-harmonic

- Fitting the time response with a linear analytical solution using optimization [5]
  - Expansion of work done by Goyder et al. [6]
  - $\hat{y} = \sum_{j=1}^{M} e^{-\beta_j t}[\mathrm{A}_j \cos(\alpha_j t) - \mathrm{B}_j \sin(\alpha_j t)] + \mathrm{C}$
- Need to “window” the response such that the response within the window is linear.



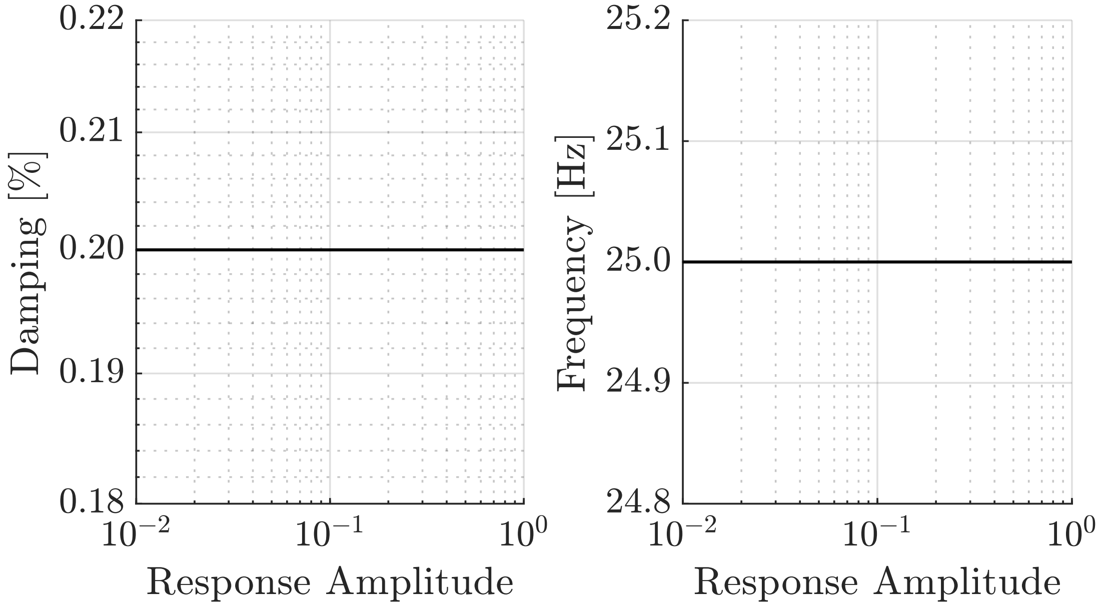


Takeaway: Direct Time Fitting can be used on multi-harmonic signals, but is highly dependent on window size and overlap ratio

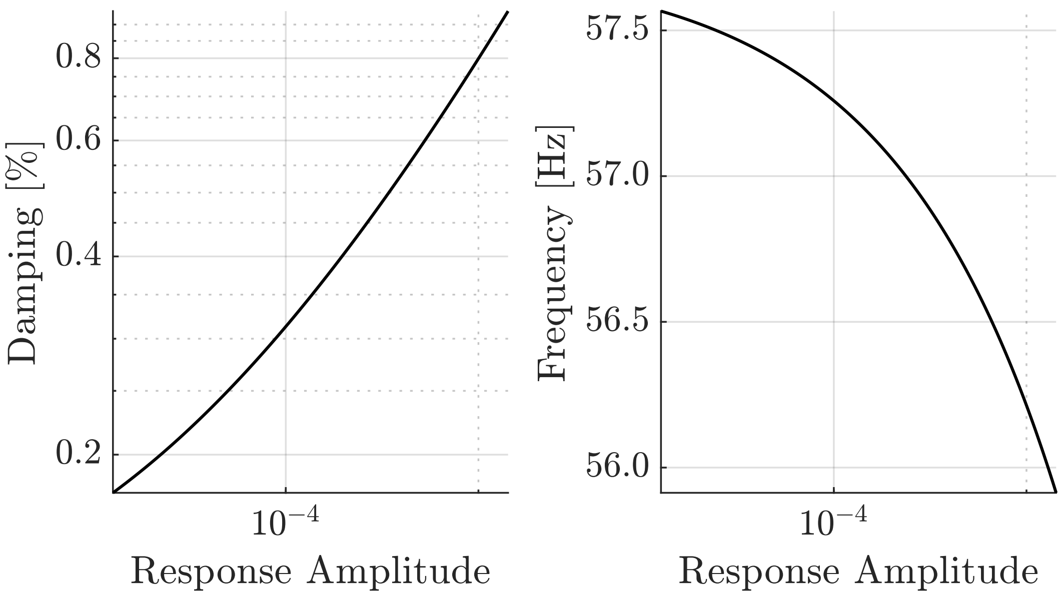
Verification of Methods

Verification on Linear and Nonlinear Responses

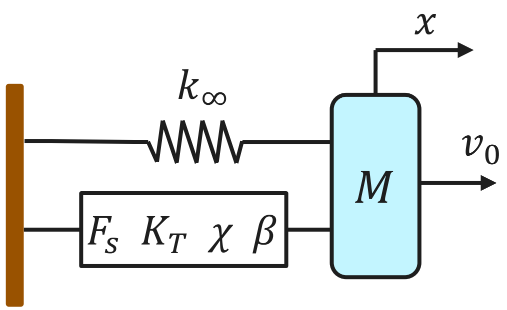
Linear response



Nonlinear response with Iwan element



Goal: Verify data processing methods against linear and nonlinear responses with analytical solutions

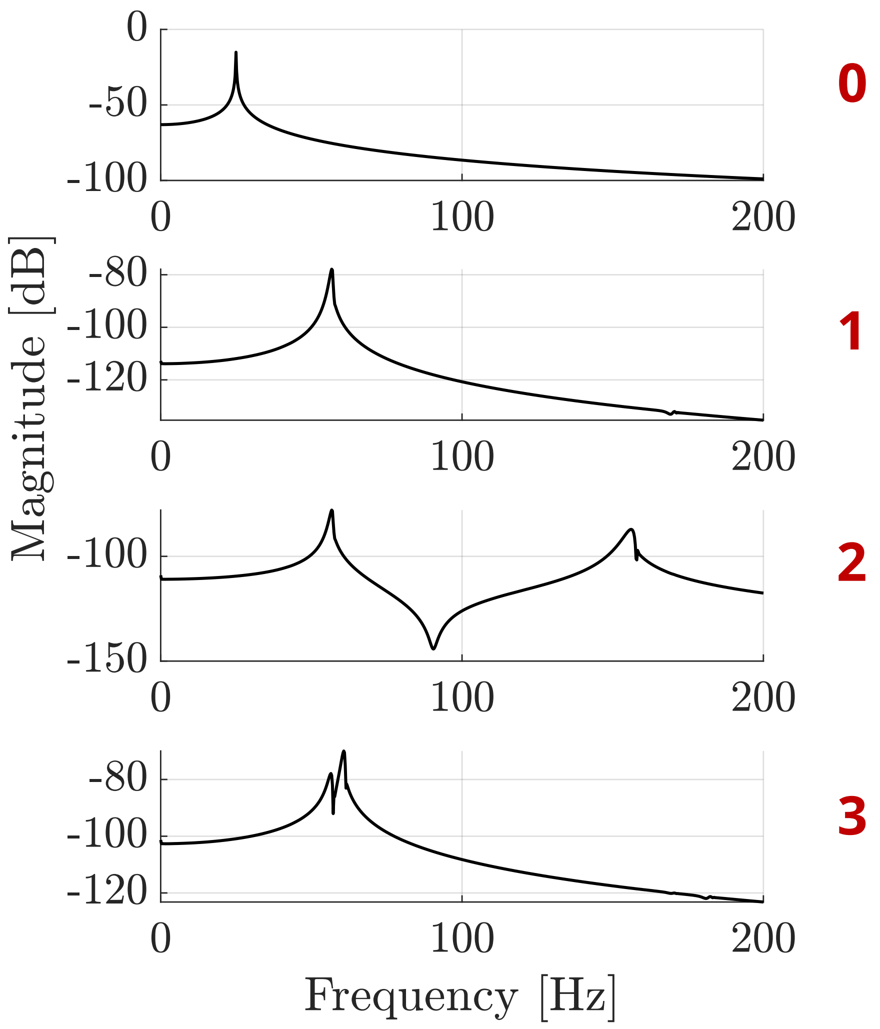


Oscillator with Iwan element

Verification Signals

Step	Response	Damping	Frequency
0	1 DOF linear	$\zeta_1 = 0.20\ \%$	$f_1 = 25.0$ Hz
1	1 DOF Iwan	$\zeta_1 \approx 0.33\ \%$	$f_1 \approx 57.2$ Hz
2	2 DOF Iwan distant modes	$\zeta_1 \approx 0.33\ \%$ $\zeta_2 \approx 0.28\ \%$	$f_1 \approx 57.2$ Hz $f_2 \approx 158.3$ Hz
3	2 DOF Iwan close modes	$\zeta_1 \approx 0.33\ \%$ $\zeta_2 \approx 0.35\ \%$	$f_1 \approx 57.2$ Hz $f_2 \approx 61.3$ Hz

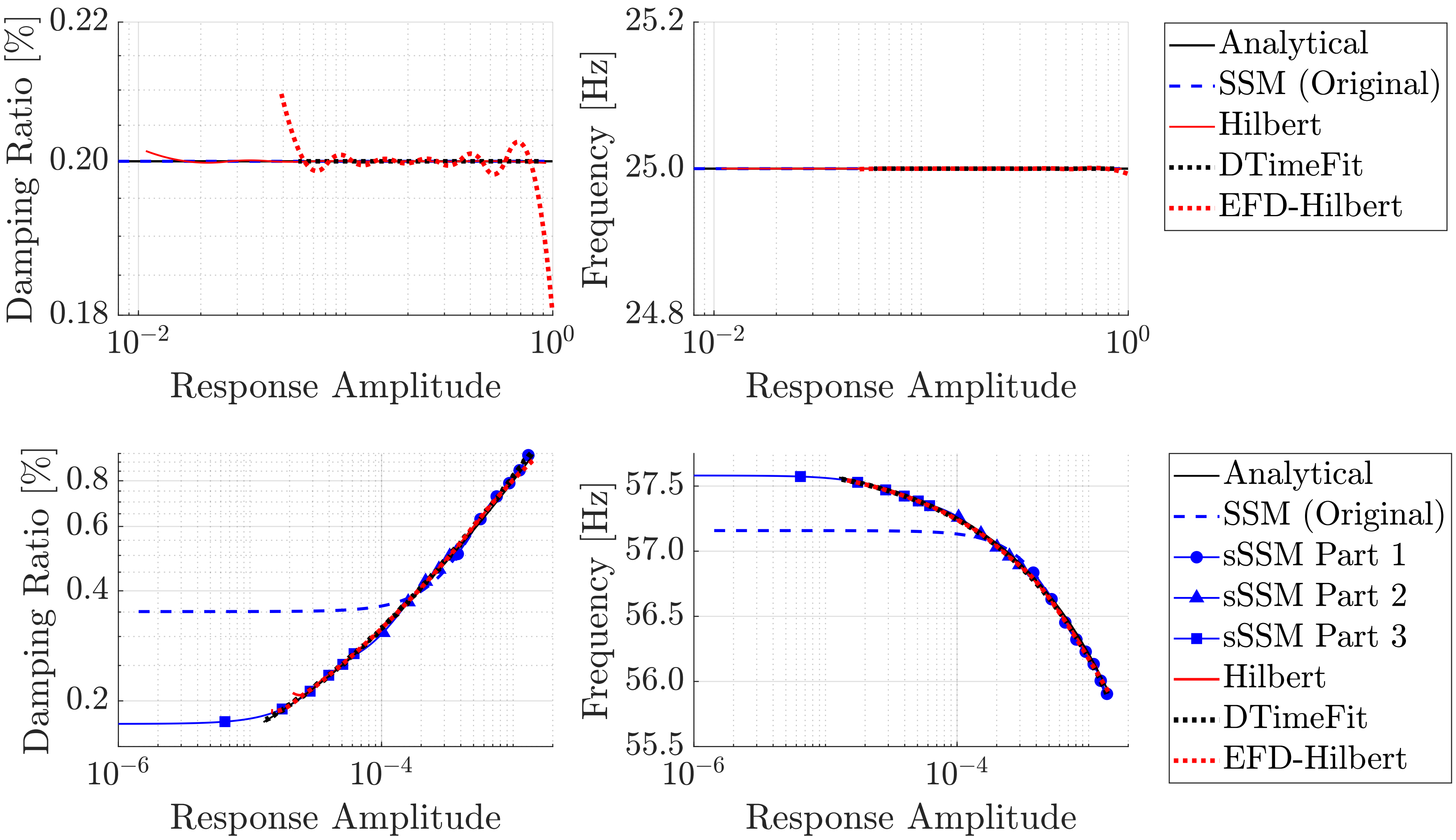
Fourier Transforms



1 DOF Responses

Linear: 25.0 Hz

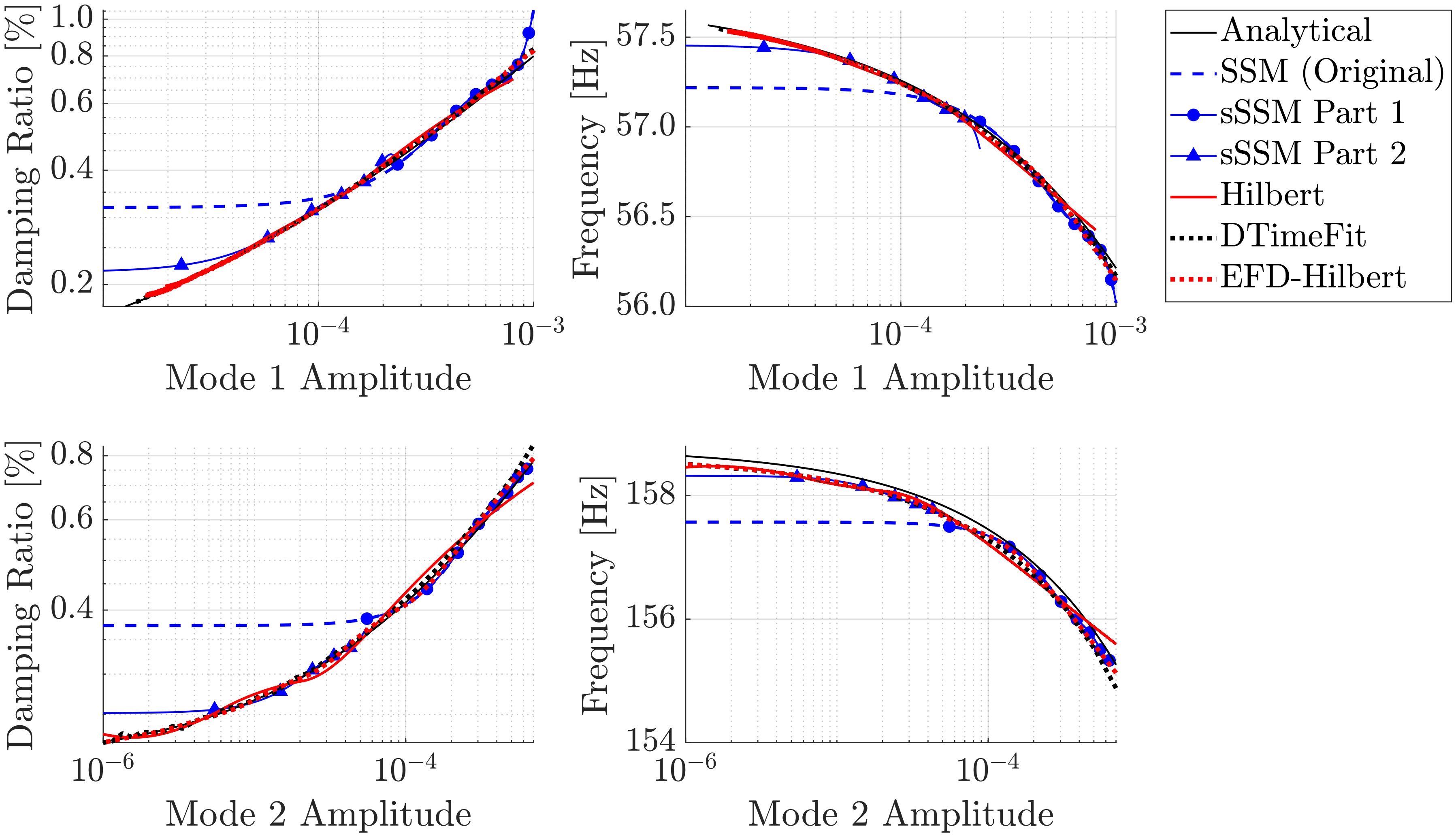
Iwan: 57.2 Hz



2 DOF Distant Modes

Mode 1: 57.2 Hz

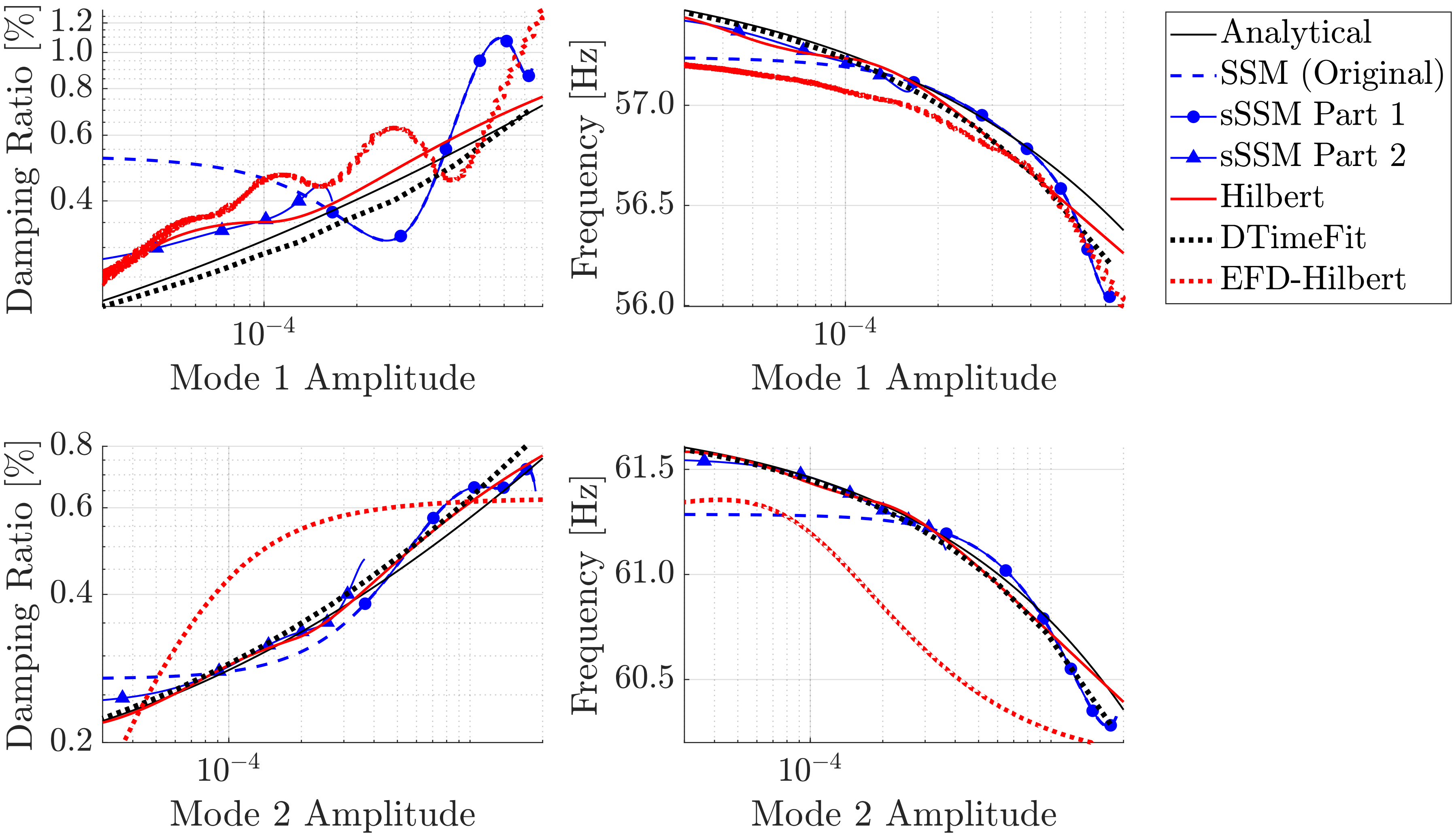
Mode 2: 158.3 Hz



2 DOF Close Modes

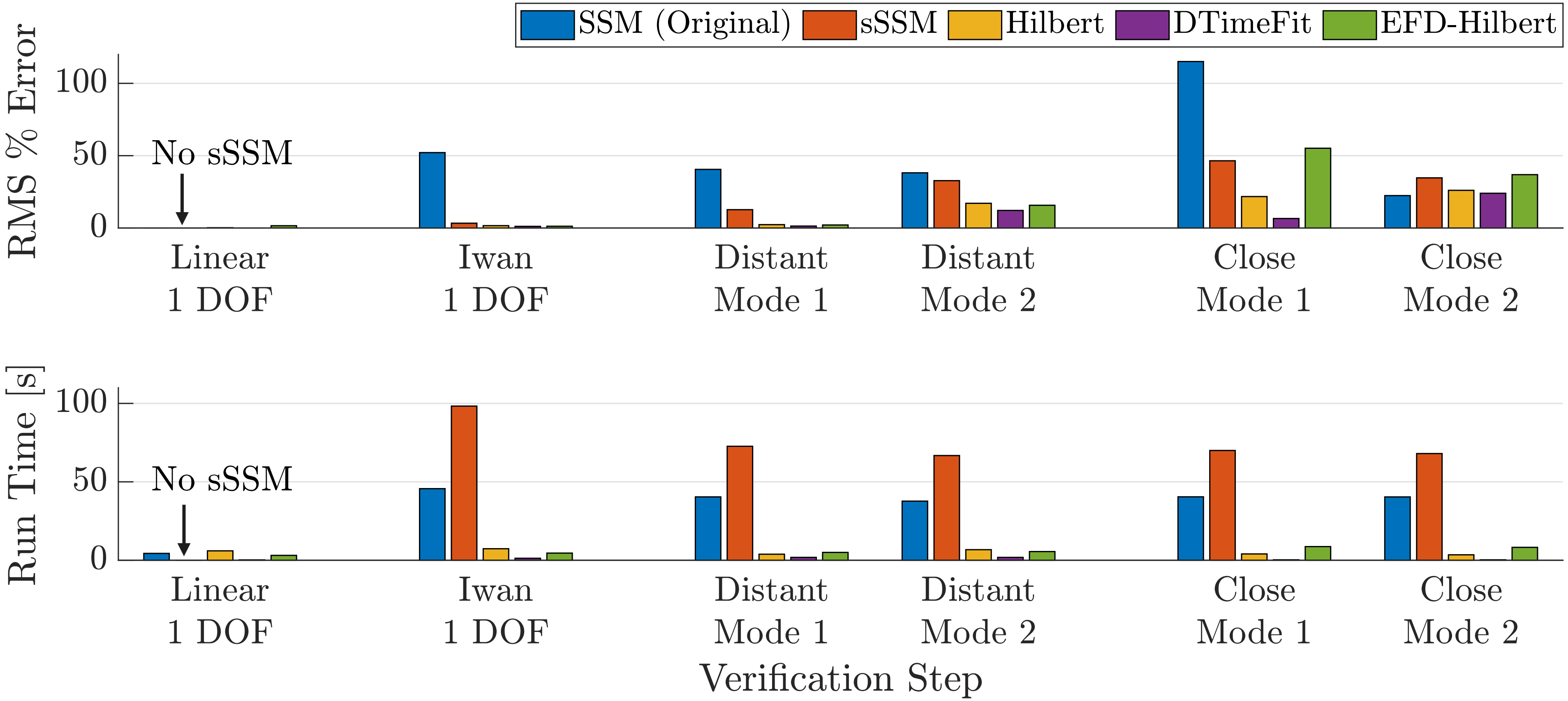
Mode 1: 57.2 Hz

Mode 2: 61.3 Hz



Verification Summary

$$\varepsilon_\omega = \text{RMS}\left(\frac{\omega_{model} - \omega_{exp}}{\omega_{exp}} \cdot 100\right), \quad \varepsilon_\zeta = \text{RMS}\left(\frac{\zeta_{model} - \zeta_{exp}}{\zeta_{exp}} \cdot 100\right), \quad \varepsilon_{Total} = \sqrt{\varepsilon_\omega^2 + \varepsilon_\zeta^2}$$



Takeaway: Direct Time Fitting was the fastest and most accurate at capturing natural frequency and damping

Application to Experimental Data

Collaborators

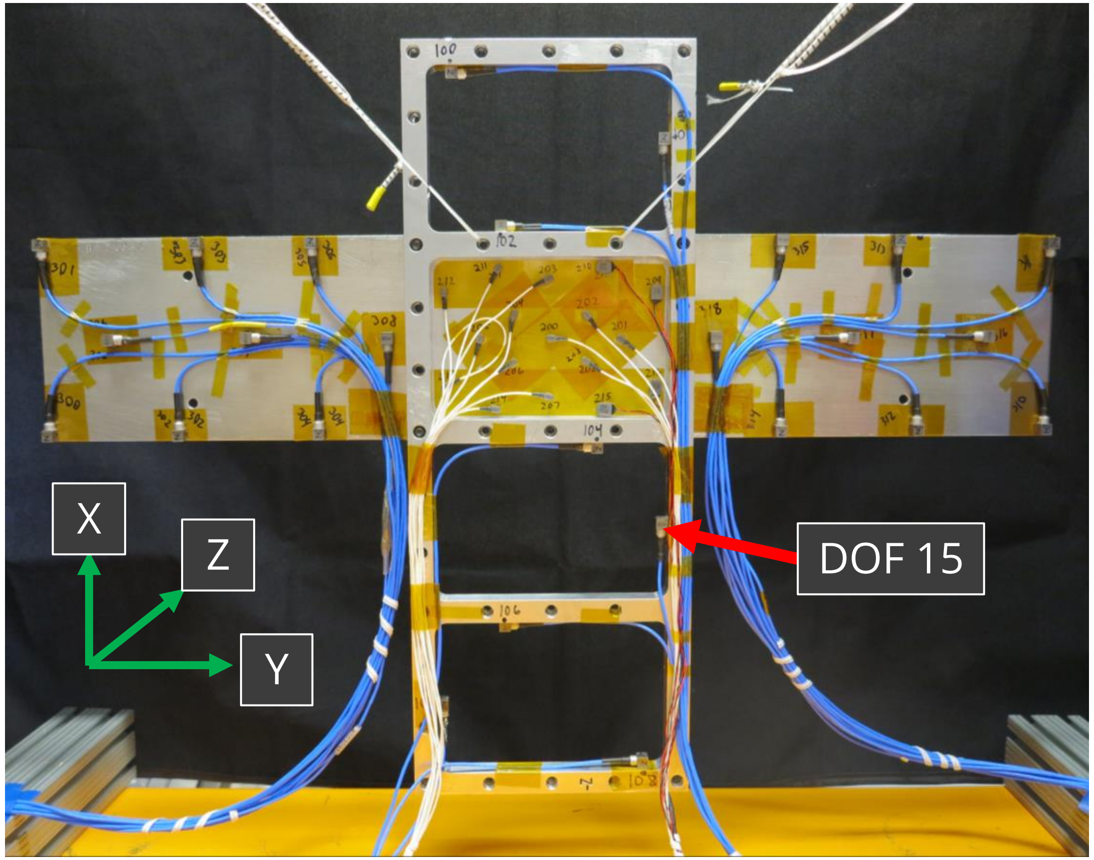
- This project is a collaboration between NOMAD, the Tribomechadynamics Research Camp (TRC), and E-TEST
- E-TEST provided the test data that was then processed using techniques chosen by NOMAD and TRC

Experimental Airplane

- "Black box" data collected January 2023
  - 95 degrees of freedom (DOF)
  - 43 nodes
- Nonlinear time series
  - 4 drive points
  - 31 impacts at various force levels
- DOF 15 – Analyzed acceleration point
  - Took an FFT of each DOF
  - Modes of interest were the most distinguished in DOF 15 data
- Modal filter + bandpass filter applied to isolate modal responses

Goal: Apply finalized methods to a structure with no analytical solution

16″

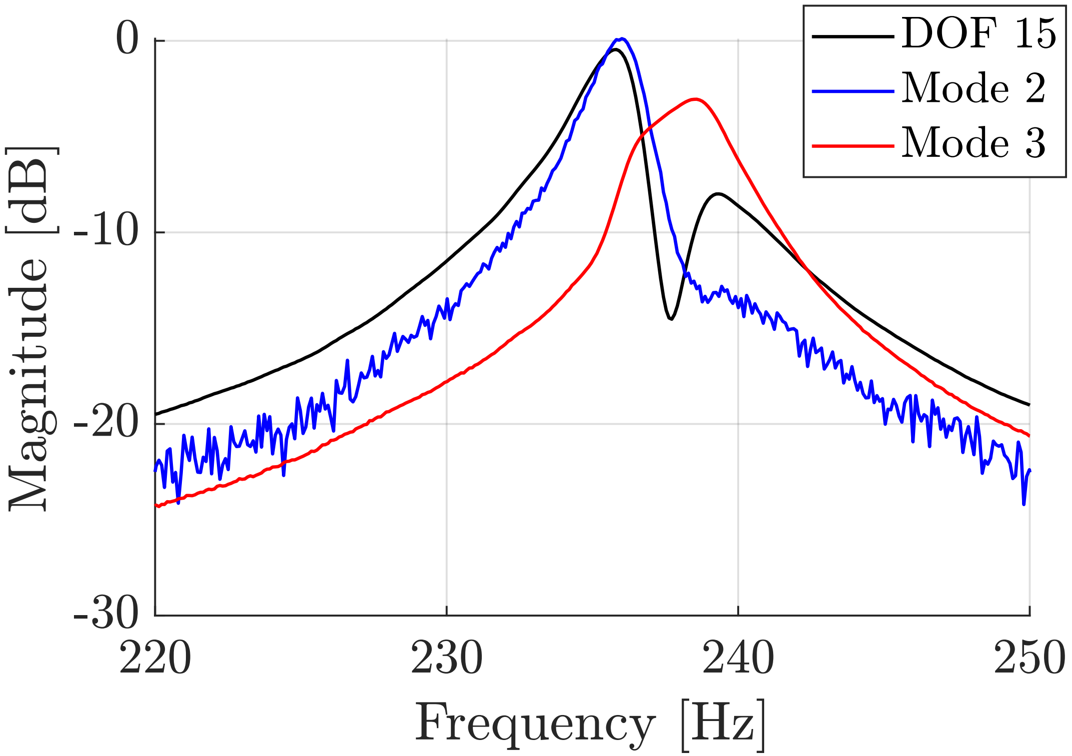


22″

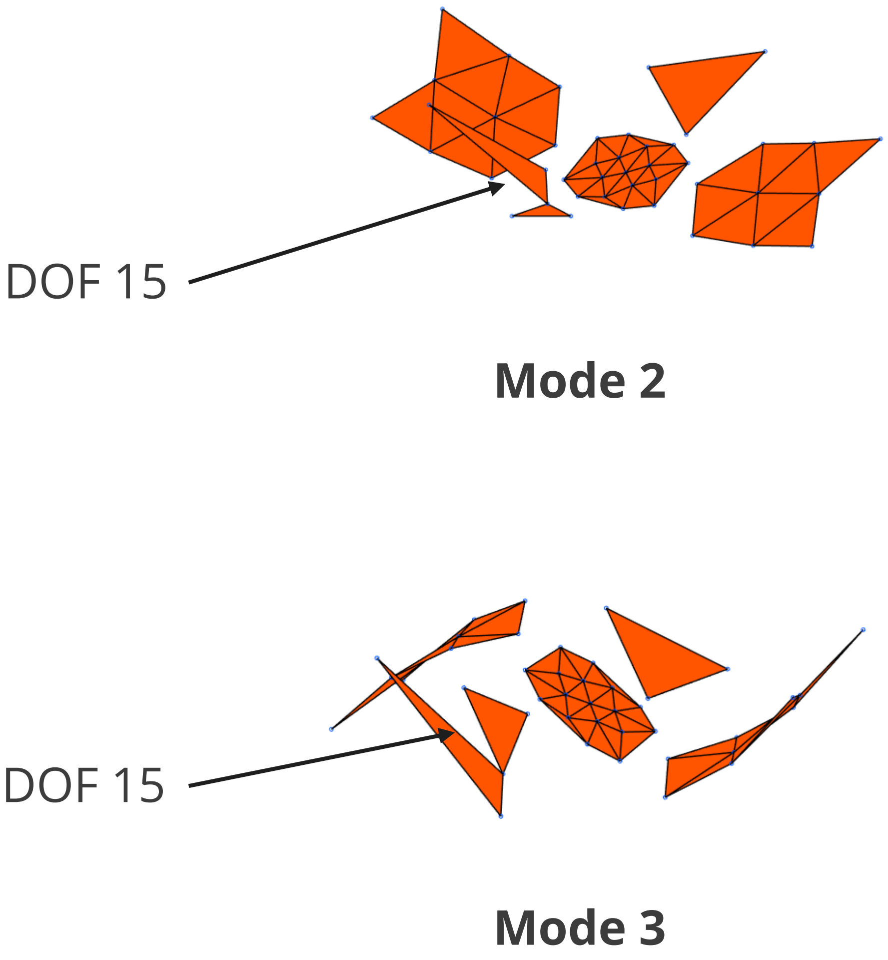
Airplane suspended from chords to simulate free boundary conditions

Two Closely Spaced Modes

Modes 2-3: $\Delta f_n = 1.4$ Hz



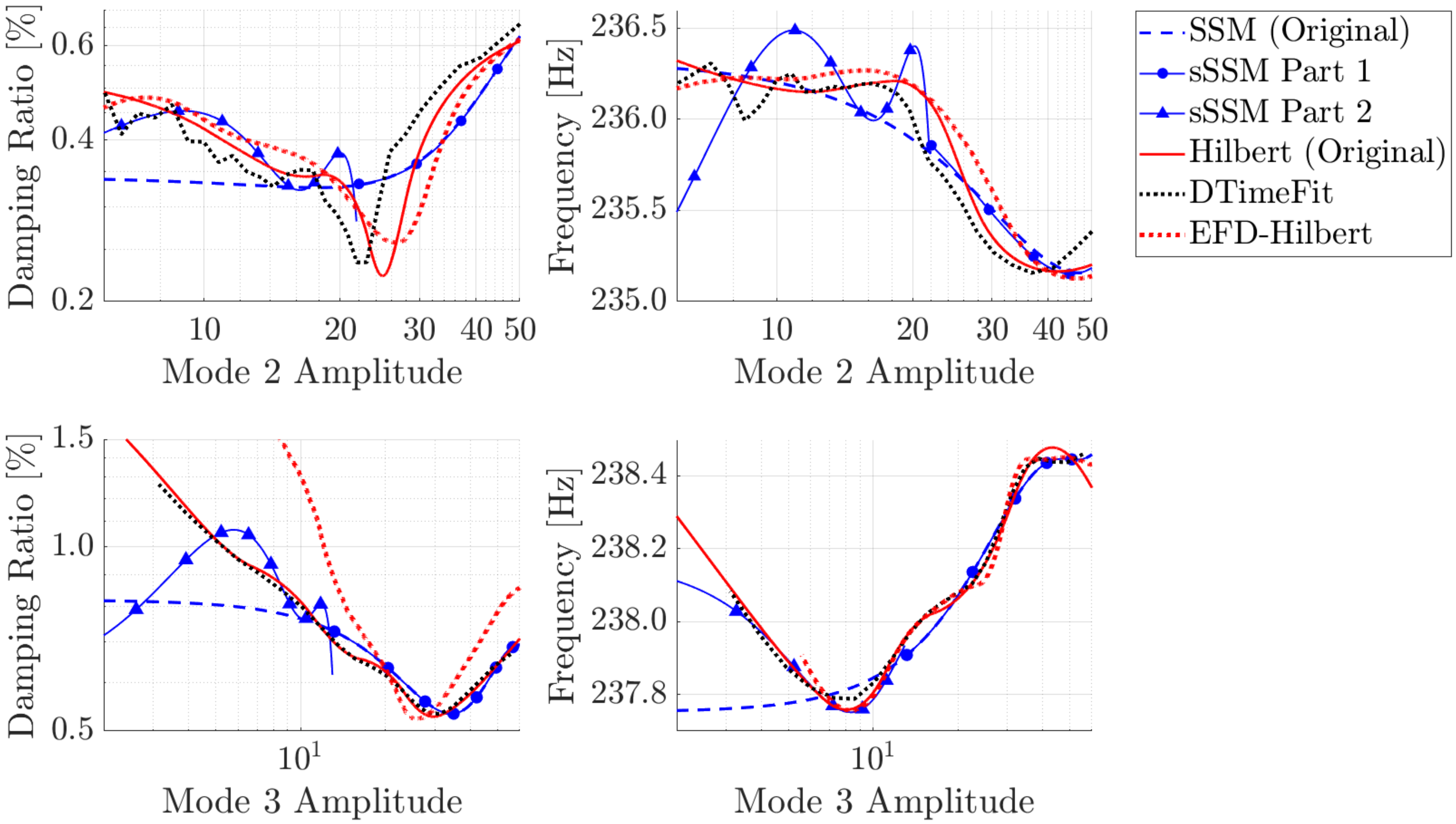
FFT of modes 2-3 and full response at DOF 15



Two Closely Spaced Modes

Mode 2: 236.2 Hz

Mode 3: 237.6 Hz



Conclusions and Future Work

Conclusions

- **Direct Time Fitting – Improved computational speed and accuracy**
  - Quick and effective.
  - Difficult to determine window size and overlap ratio.
- **Spectral Submanifolds – Powerful reduced-order modeling tool that works best at large amplitudes**
  - Trade-off between generalizability and capturing the low amplitude features of a single trajectory
- **Empirical Fourier Decomposition – Preprocessing for higher order quantitative analysis**
  - Better isolate monoharmonic responses from multiharmonic data
  - Need to fine tune parameters to avoid mis-quantifications of damping ratio

Future Work

- **Direct Time Fitting – Automating window sizing**
  - Adaptive window size based on fitting a certain amount of cycles for the frequency being fit
  - Quick sortesque windowing
    - Split the response in two. Solve for each. Compare to an error metric. Split the window again if the fit doesn't meet requirement.
- **SSMs - Improving accuracy at low amplitudes**
  - Fit a single reduced-order model on a log scale rather than linear scale
  - Stitch multiple reduced-order models together from split SSMLearn
- **EFD – Integrating results with machine learning and other advanced methods**
  - Leverage for construction of a “white box" machine learning (ML) architecture
    - Quantify mathematical building blocks and interactions
  - Make extracted modes representative of the physical dynamics in a system

Acknowledgements

Special thanks to our mentors, Aabhas Singh, Matthew Allen, Matthew Brake, Robert Kuether, Benjamin Moldenhauer, Daniel Roettgen, and Kevin Dowding, for all the time they have spent helping us throughout this project.

This research was conducted at the 2024 Nonlinear Mechanics and Dynamics Research Institute hosted by Sandia National Laboratories and the University of New Mexico.

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References

[1] Maji, Uday, and Saurabh Pal. "Empirical mode decomposition vs. Variational mode decomposition on ECG signal processing: A comparative study. "*2016 International conference on advances in computing, communications and informatics (ICACCI)*. IEEE, 2016.

[2] Zhou, Wei, et al. "Empirical Fourier Decomposition: An accurate signal decomposition method for nonlinear and non-stationary time series analysis." *Mechanical Systems and Signal Processing* 163 (2022): 108155

[3] M. Feldman, "Non-linear system vibration analysis using Hilbert transform--I. Free vibration analysis method 'Freevib,'" *Mech. Syst. Signal Process*., vol. 8, no. 2, Art. no. 2, 1994.

[4] Moldenhauer, Benjamin John. *Nonlinear System Identification Methods for Characterizing Amplitude Dependent Modal Properties*. Diss. University of Wisconsin--Madison, 2022.

[5] Singh, Aabhas, and Matthew S. Allen. *Simultaneous Direct Time Fitting of a Multi-Mode Response to Determine the Instantaneous Frequency and Damping*. No. SAND2022-14542C. Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 2022.

[6] Goyder, Hugh GD, and Damien PT Lancereau. "Methods for the Measurement of Non-linear Damping and Frequency in Built-up Structures." *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. Vol. 58226. American Society of Mechanical Engineers, 2017.

[7] M. Cenedese, J. Axås, B. Bäuerlein, K. Avila and G. Haller. Data-driven modeling and prediction of non-linearizable dynamics via spectral submanifolds. *Nature Communications*, 13 (2022) 872.