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# GRAPH ATTENTION EMBEDDINGS AS A CAUSAL LENS IN TEMPORAL LINK PREDICTION

Presented by: Sarah Simpson

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MLDL Workshop 2024



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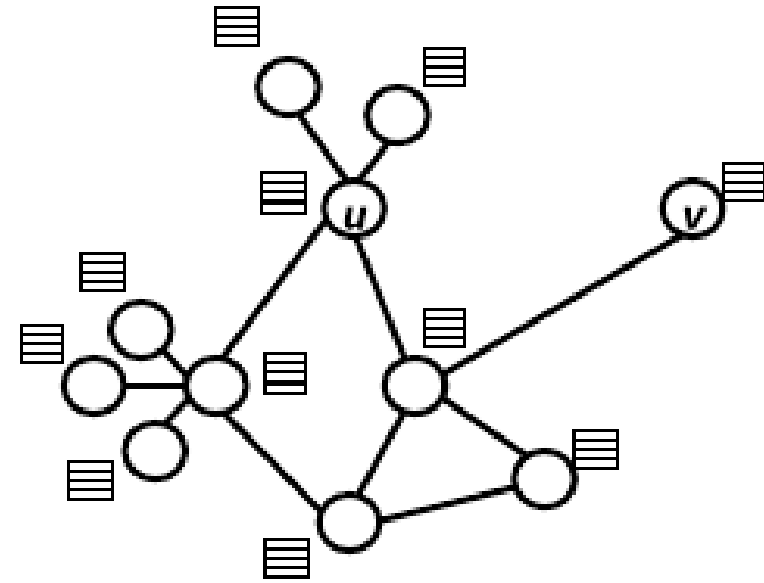
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# Background and Motivation

# TEMPORAL ATTRIBUTED GRAPHS AND CRITICAL SYSTEMS

Temporal graphs are an extensible structure that is heavily used to represent complex dynamical systems across many high impact domains.



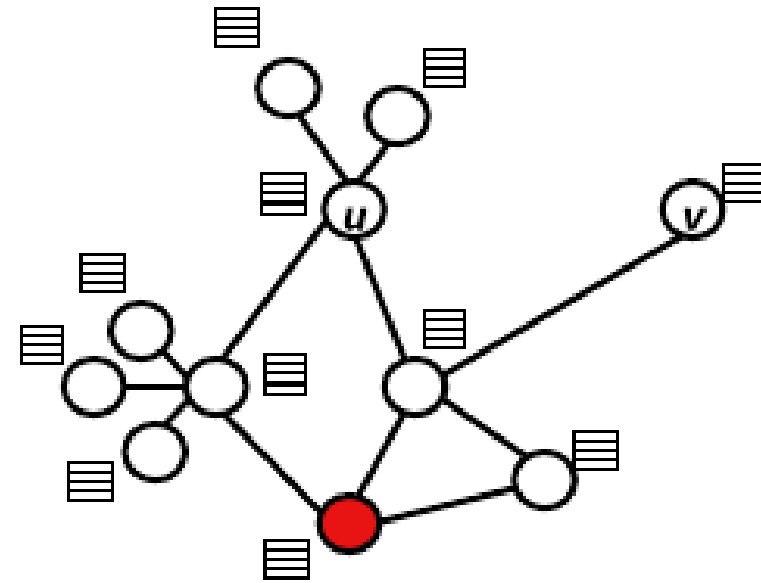
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Common tasks on these systems include

- **Node classification**
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## Example applications

Identify bad actors in social systems

Find vulnerable components in an electric grid

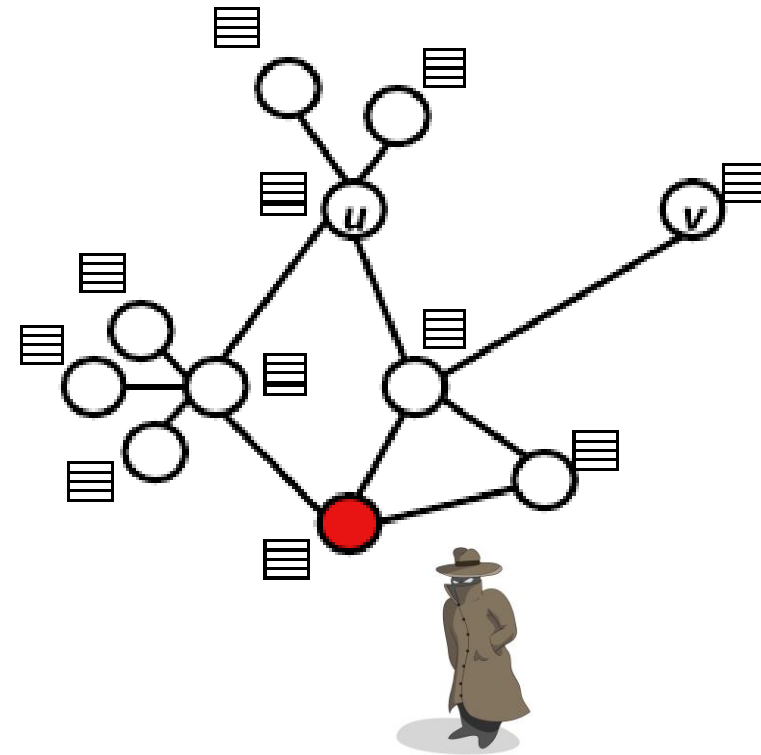
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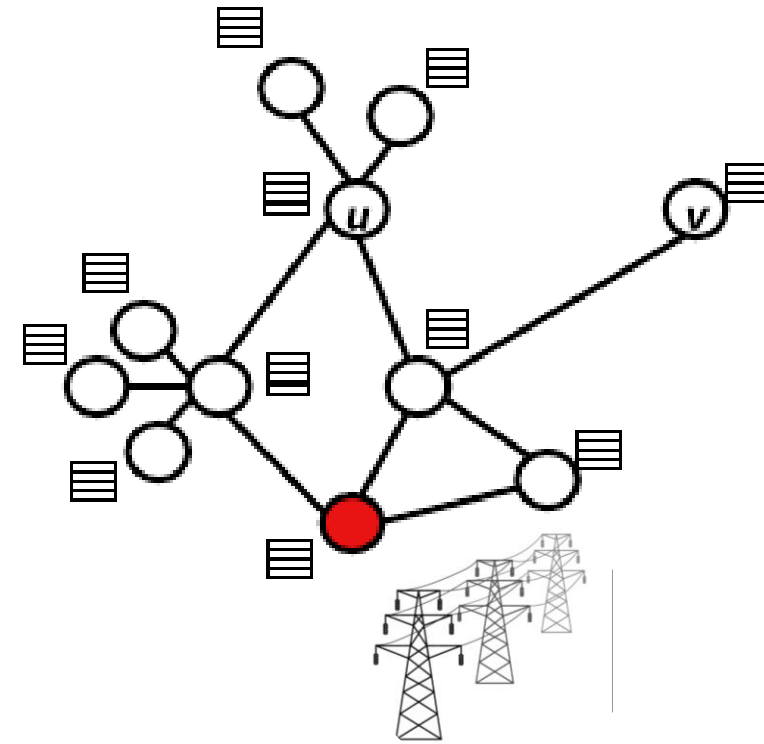
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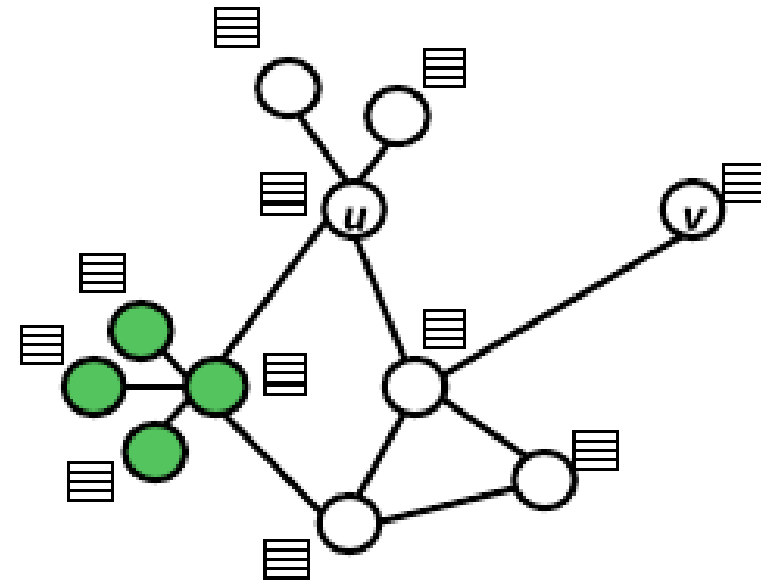
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Identify vulnerable populations for pandemic intervention and countermeasure delivery

Disinformation and influence campaign identification

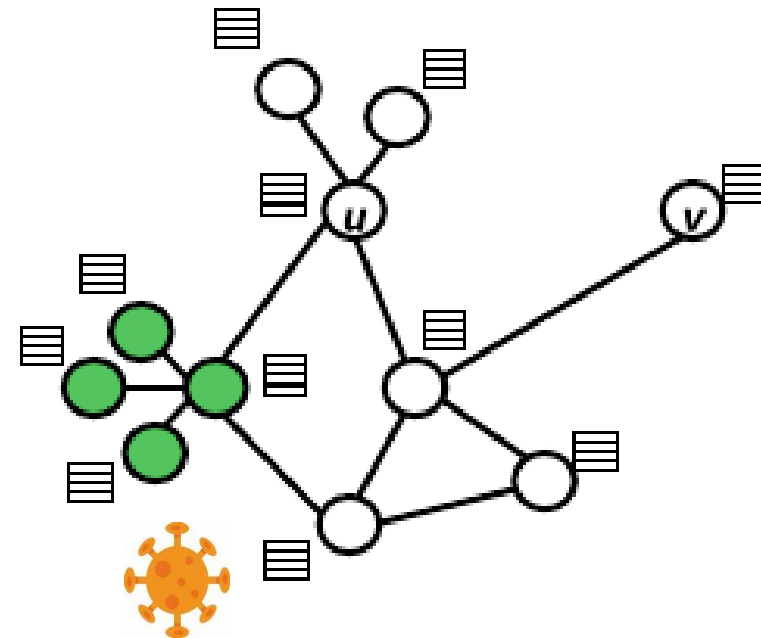
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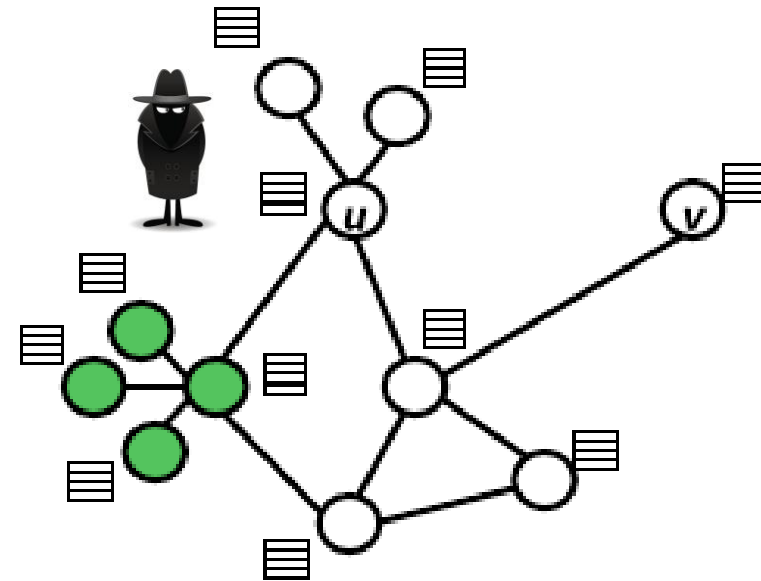
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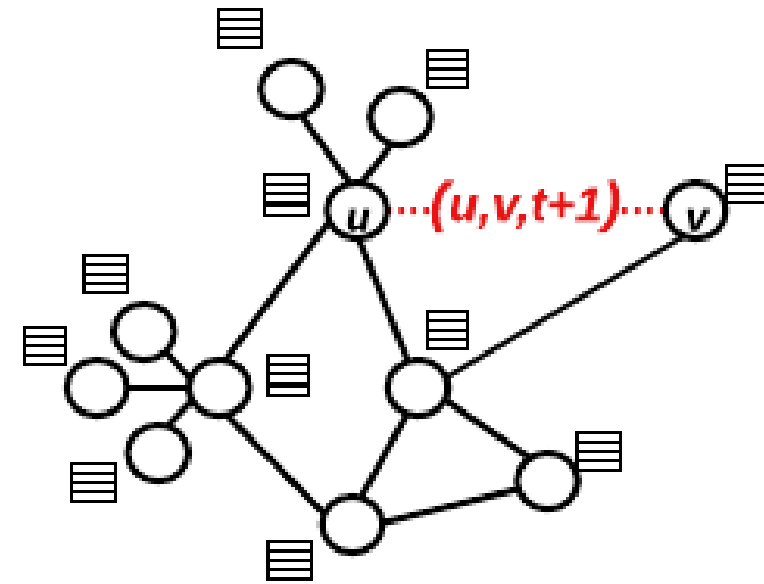
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## Example applications

Predict cascading failures in electric grid systems

Anticipate high impact scientific collaborations

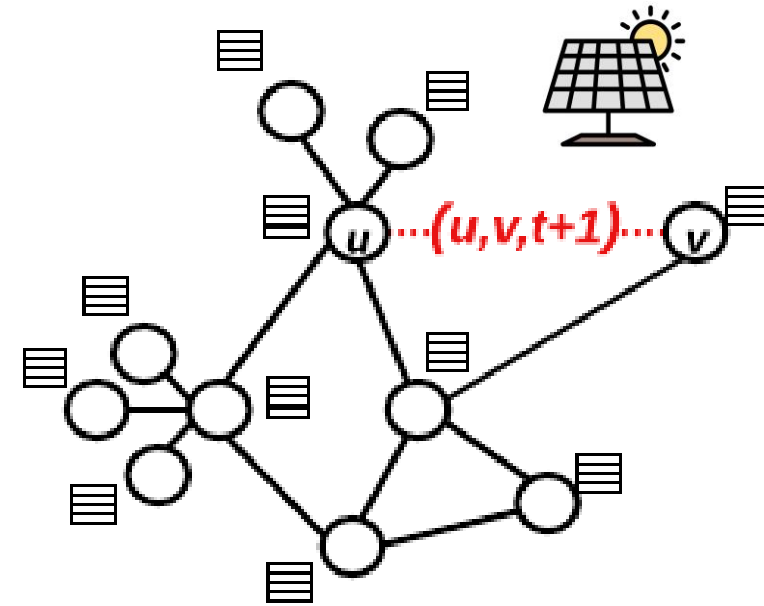
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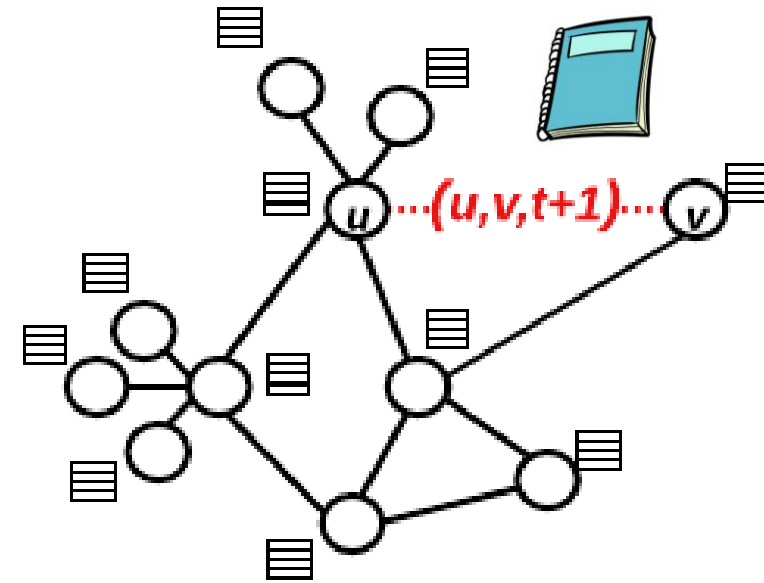
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# TEMPORAL ATTRIBUTED GRAPHS AND CRITICAL SYSTEMS

These representations are used for far more than advertising, and marketing, social media. They are frequently used in high consequence modeling applications.



Cyber networks



Global trade



Transportation and logistics

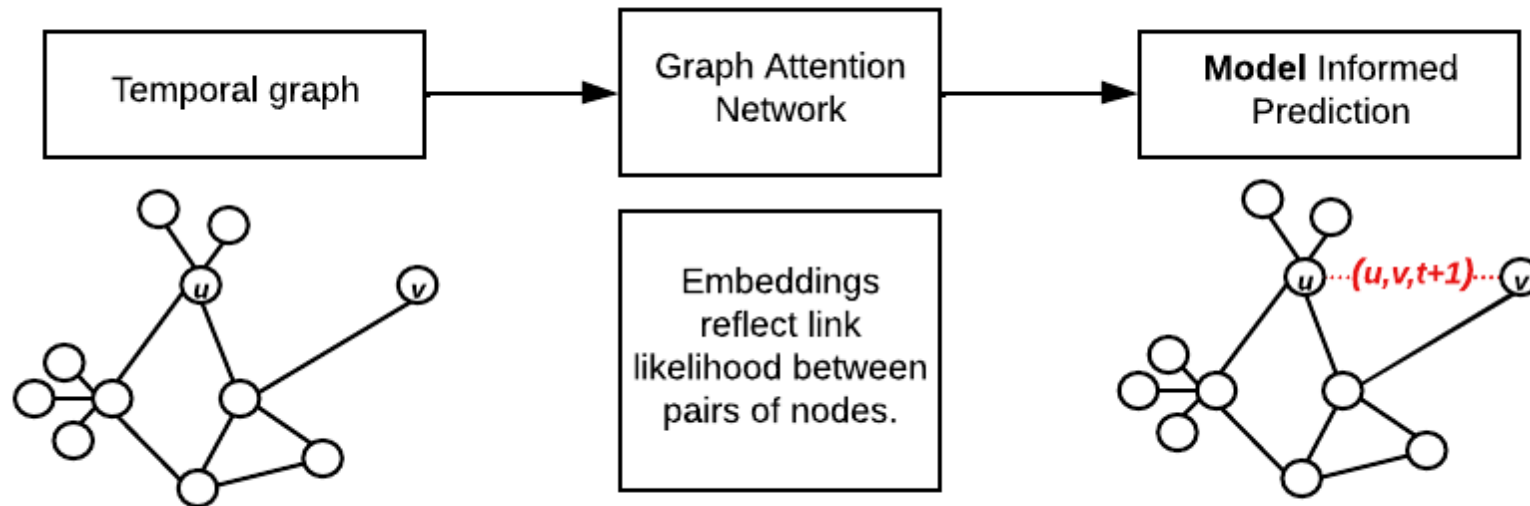
Social networks



# CURRENT MODELING STATE OF THE ART



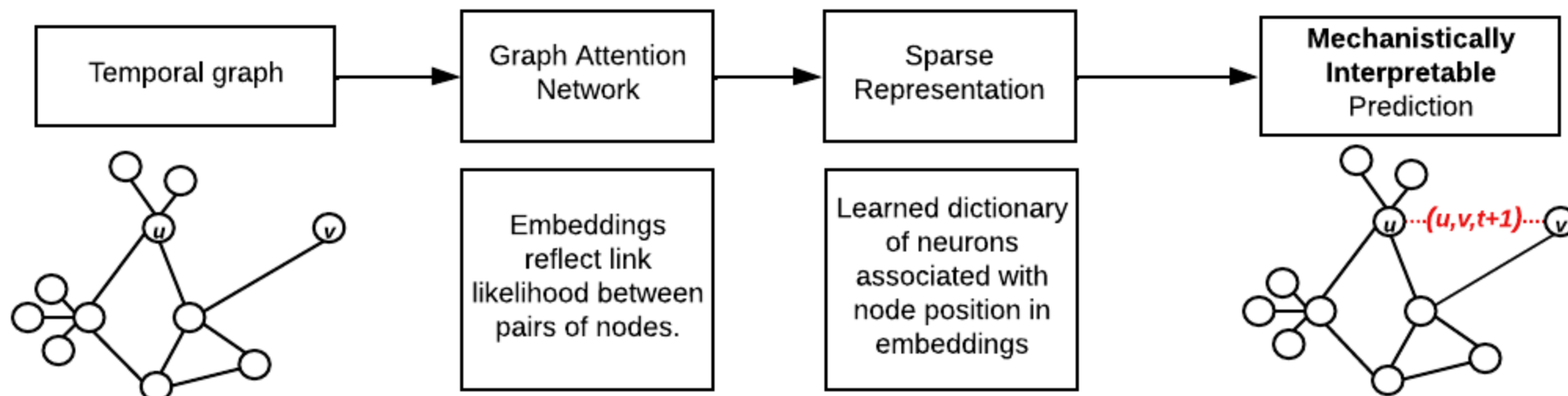
Current state-of-the-art algorithms are performant but lack causal structure and explainability. This can preclude their use in higher consequence modeling and decision making.





# INCORPORATING MECHANISTIC INTERPRETABILITY

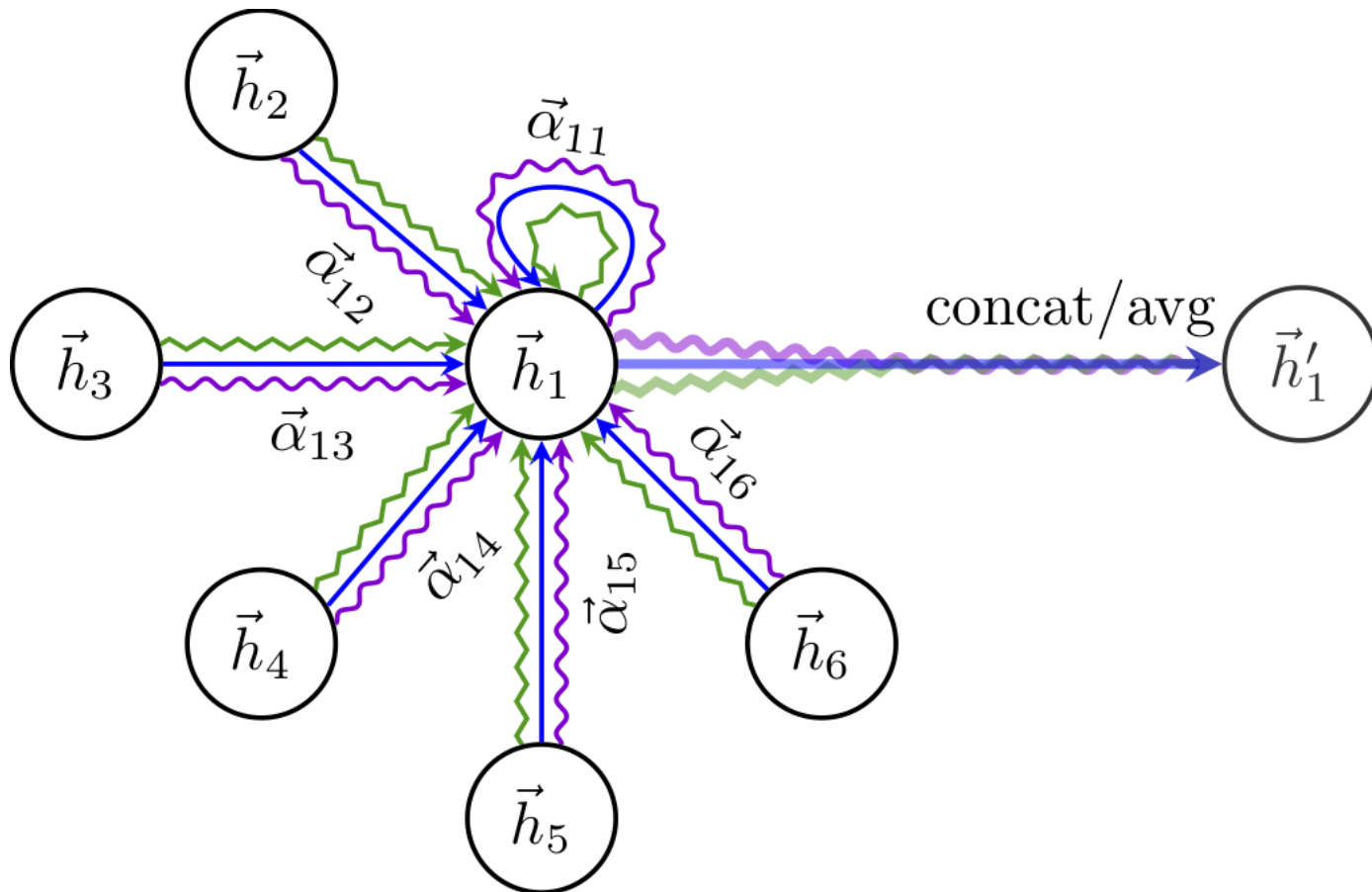
We seek to develop **algorithm agnostic** methods to incorporate interpretable, causal explanations into graph prediction models using the latent representations that underpin model inference.



# LATENT REPRESENTATIONS: EMBEDDINGS AND NODE SELF ATTENTION



Contextually rich relationships between nodes are learned through the attention mechanism.



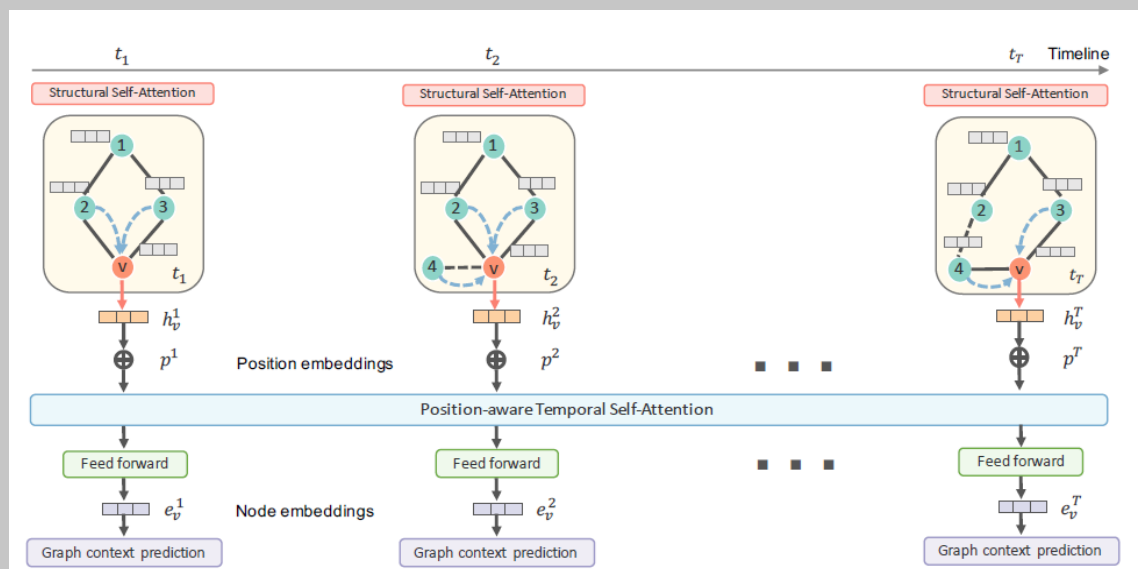
- Technique originally introduced in large language modeling
- Each node and edge embedding is associated with query, key, and value learned vectors.
- Similarity between the query of each element and the key of all other elements is used by the model to learn relationships between sentences, paragraphs, etc





## DySAT (Dynamic Self-Attention Network)

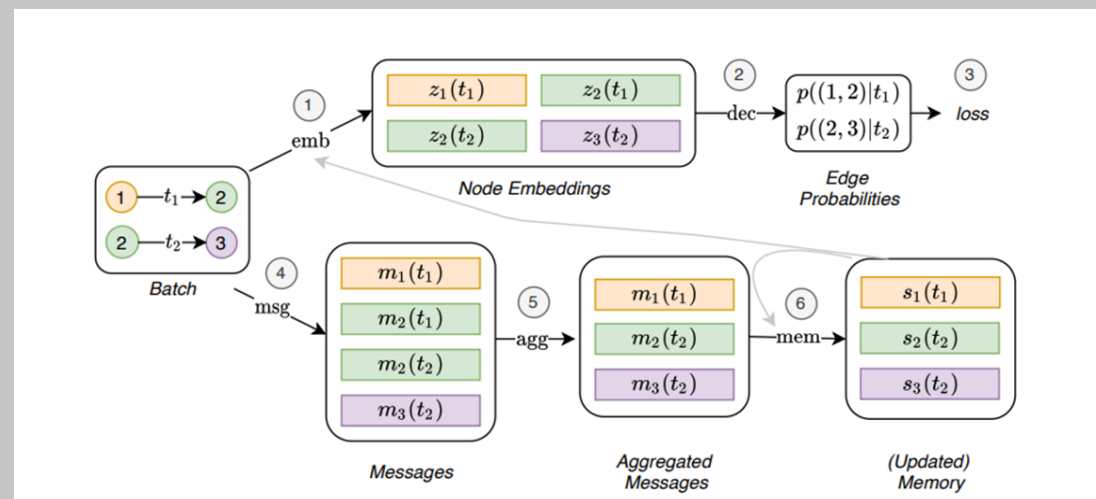
- Learns local and global dependencies through hierarchical self-attention layers
- Discrete-time dynamics



Sankar, Aravind, et al. "Dysat: Deep neural representation learning on dynamic graphs via self-attention networks." *Proceedings of the 13th international conference on web search and data mining*. 2020.

## TGN (Temporal Graph Networks)

- Adopts a message-passing framework to handle dynamic graphs
- Continuous-time dynamics

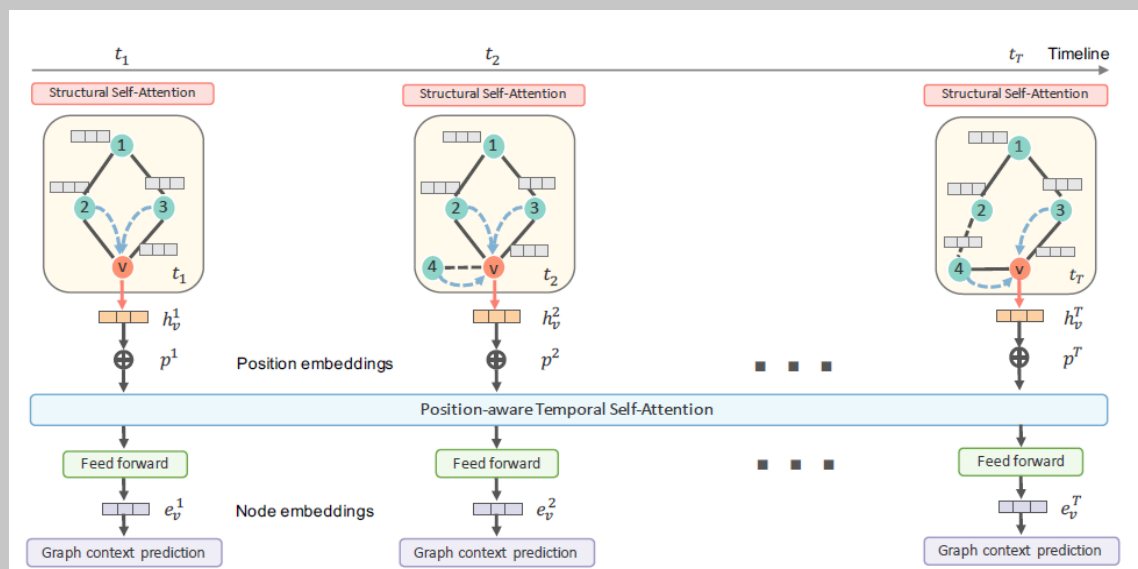


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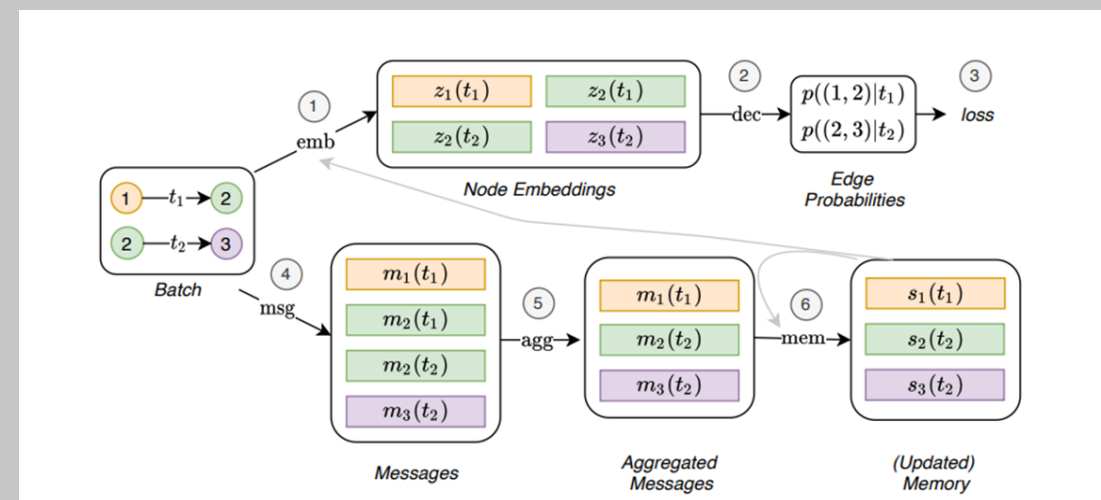
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# DYSAT ARCHITECTURE



**Input:** Adjacency matrix, node attributes

**Step 1:** Structural self-attention

**Step 2:** Encode temporal position

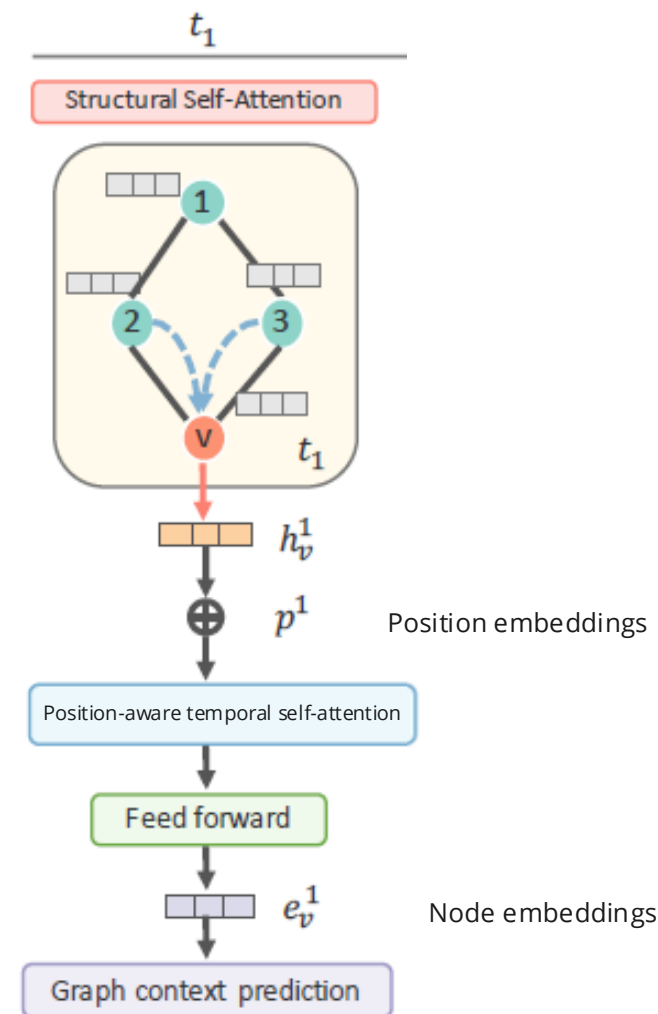
**Step 3:** Temporal self-attention

**Output:** 1 node embedding vector per node per snapshot

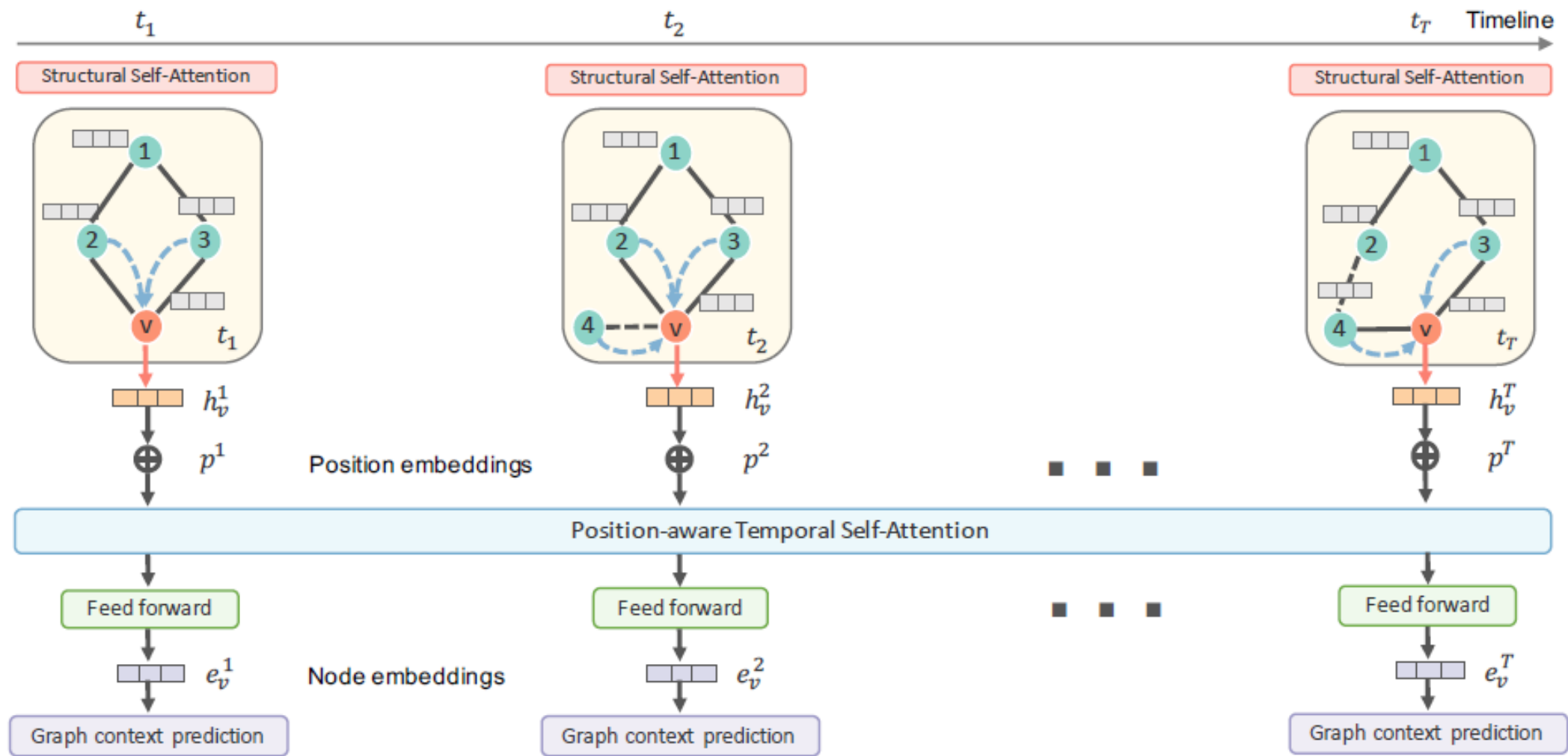
Input:  $x_u$ , adjacency matrix

$$z_v = \sigma \left( \sum_{u \in N_v} \alpha_{uv} W^s x_u \right)$$

$$Z_v = \beta_v (X_v W_v)$$



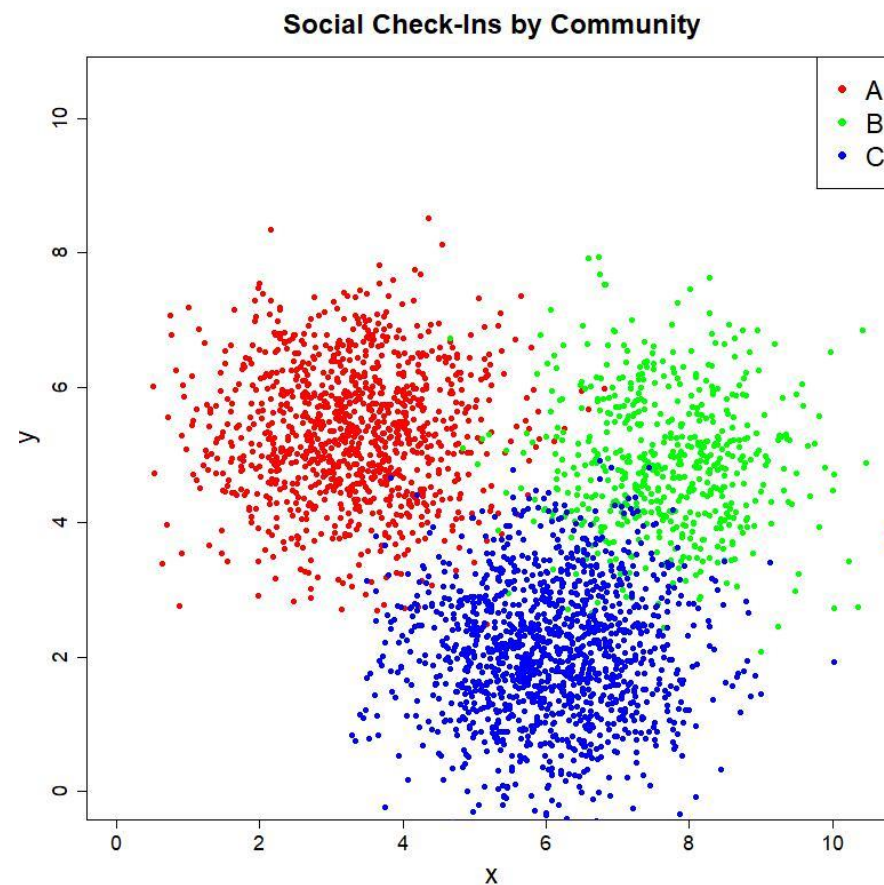
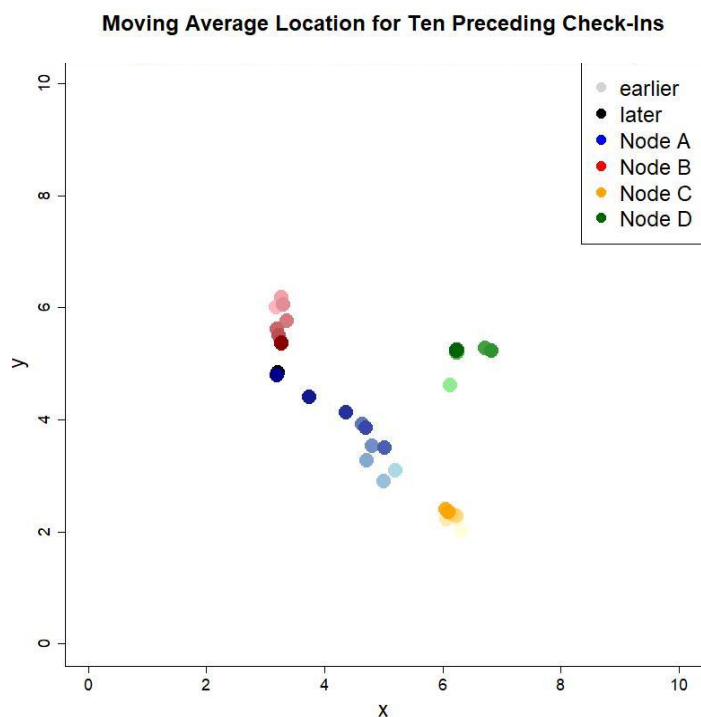
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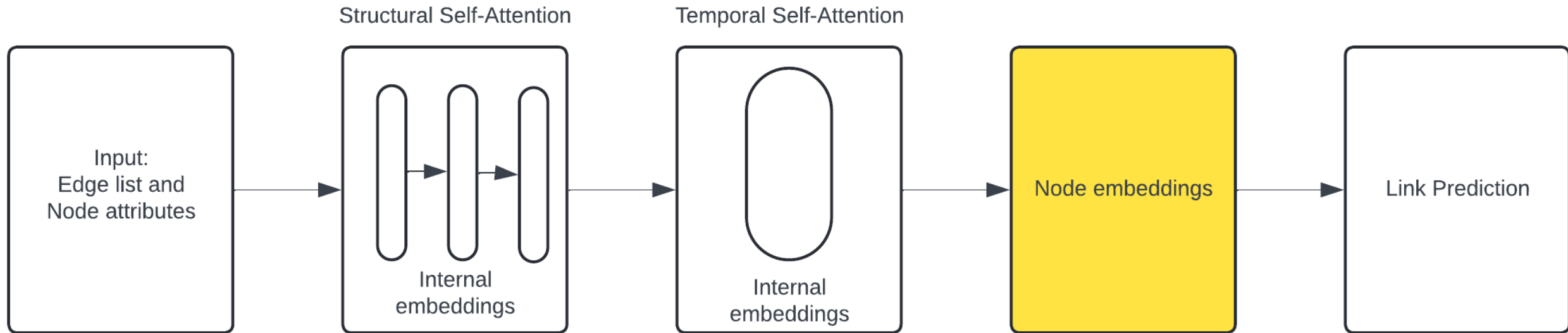


# SIMULATED SOCIAL INTERACTION EXPERIMENT

- 3 communities with locations sampled in a distribution around their center
- 30 days, broken into 10 ESSD snapshots
- 100 nodes, each belonging to 1 or more communities
  - Nodes with shared communities have positive probability of check-in
- Probability of edges varies by
  - Number of communities nodes share
  - Average location proximity



# SIMULATED SOCIAL INTERACTION EXPERIMENT



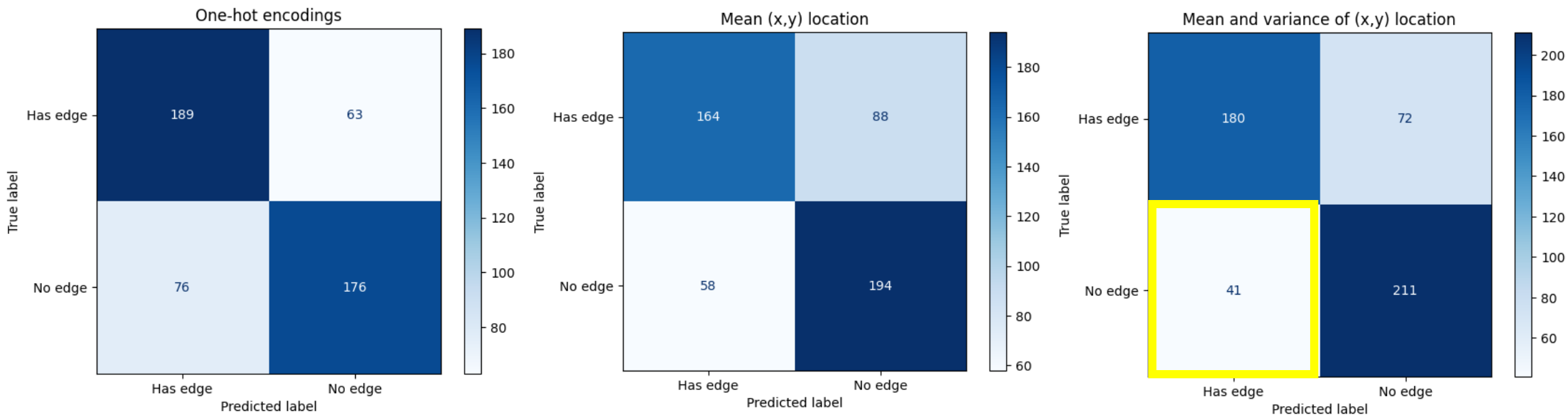


# Node Attribute Modification Experiments



# NODE ATTRIBUTE VARIATION AND MODEL PERFORMANCE

- How does modifying attributes impact model performance?
  - Number of false positive predictions decreases with addition of features which enhance underlying dataset dynamics

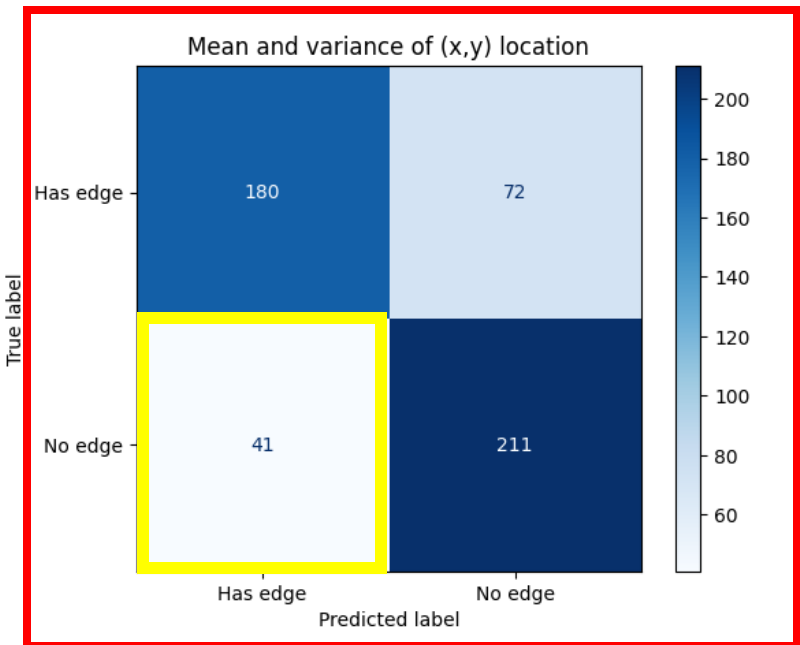
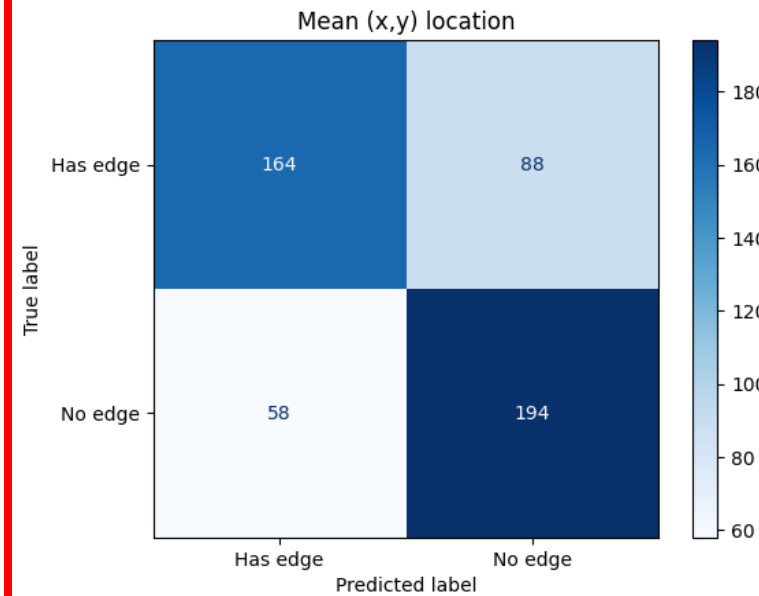
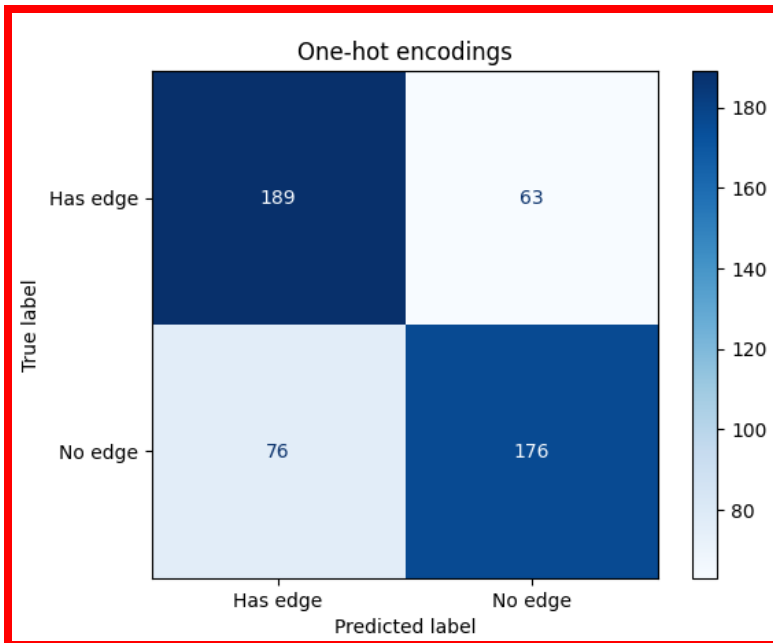




# NODE ATTRIBUTE VARIATION AND MODEL PERFORMANCE



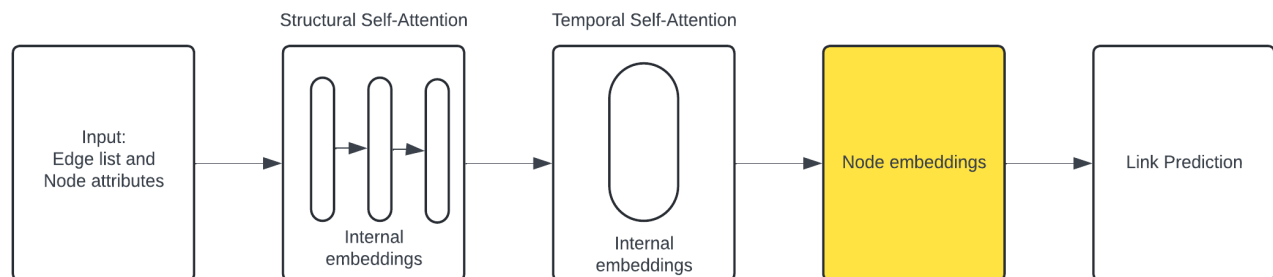
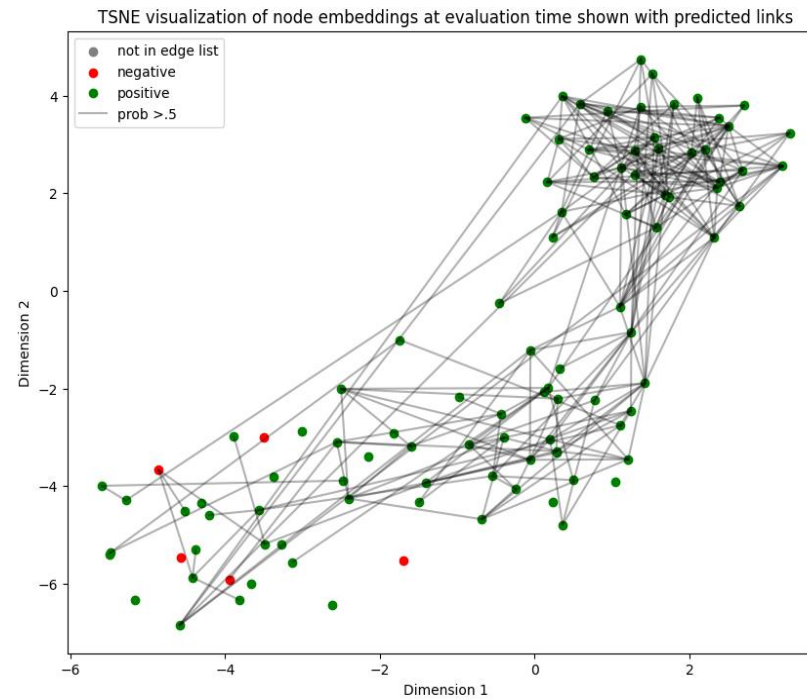
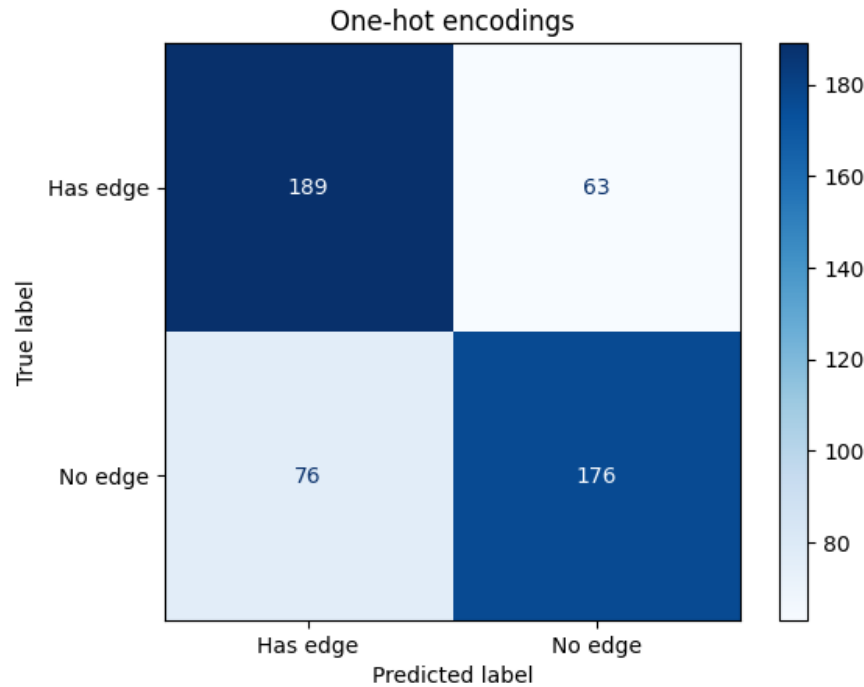
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# NODE ATTRIBUTE VARIATION AND LATENT REPRESENTATIONS



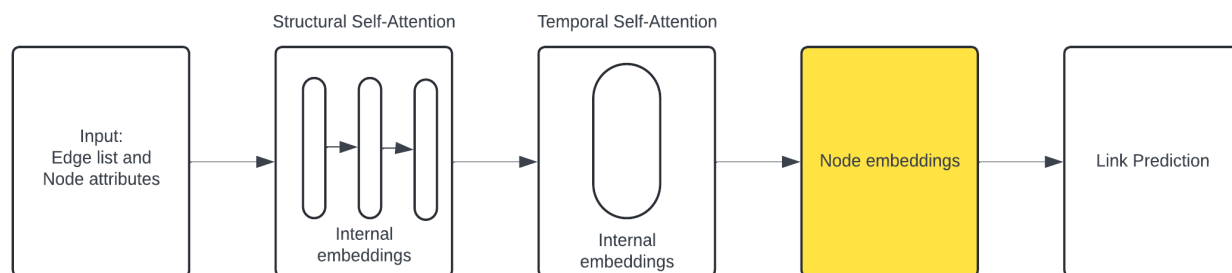
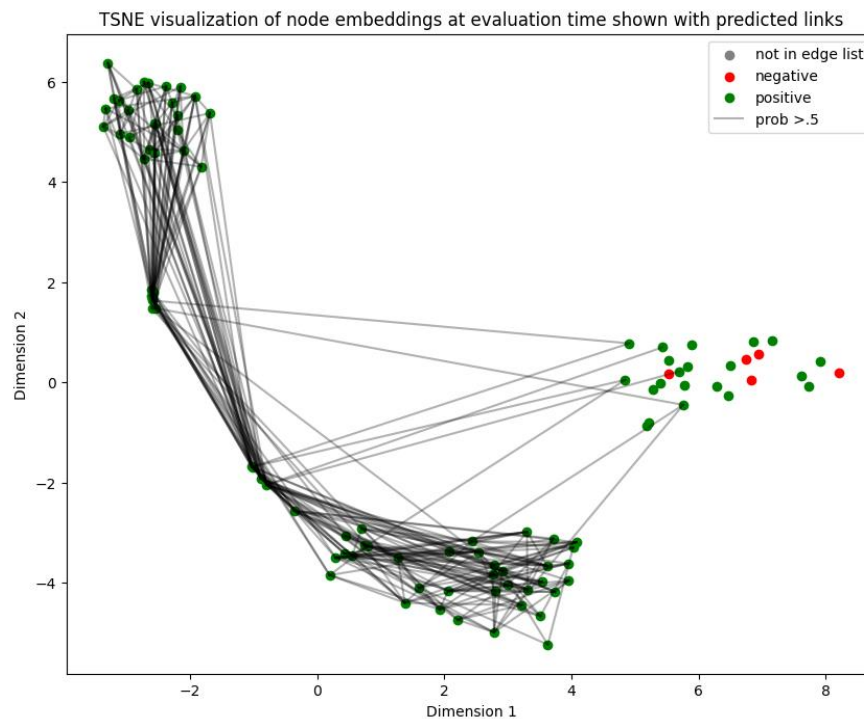
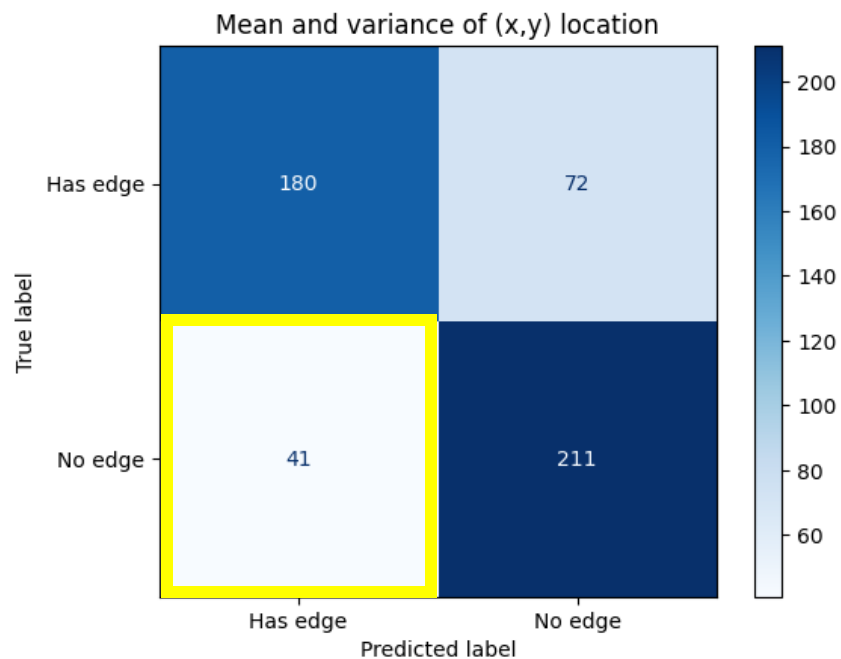
These embeddings are used by the model to predict link likelihood



# NODE ATTRIBUTE VARIATION AND LATENT REPRESENTATIONS



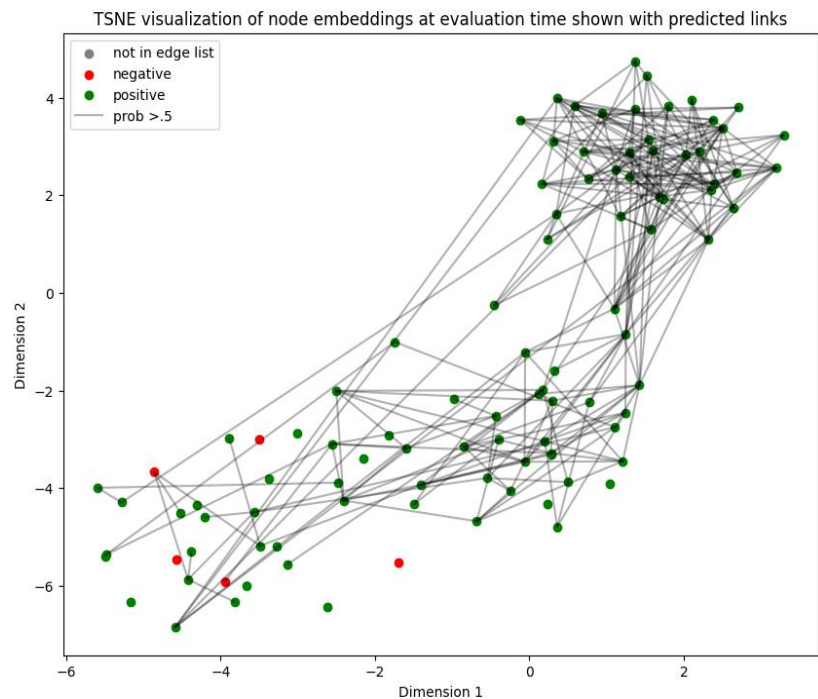
The increased attribute richness decreases false positives, and creates a more complex and separable node embedding



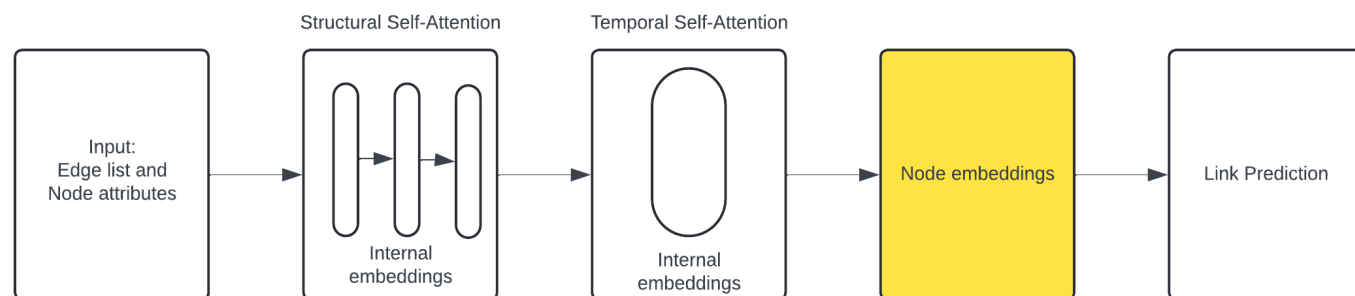
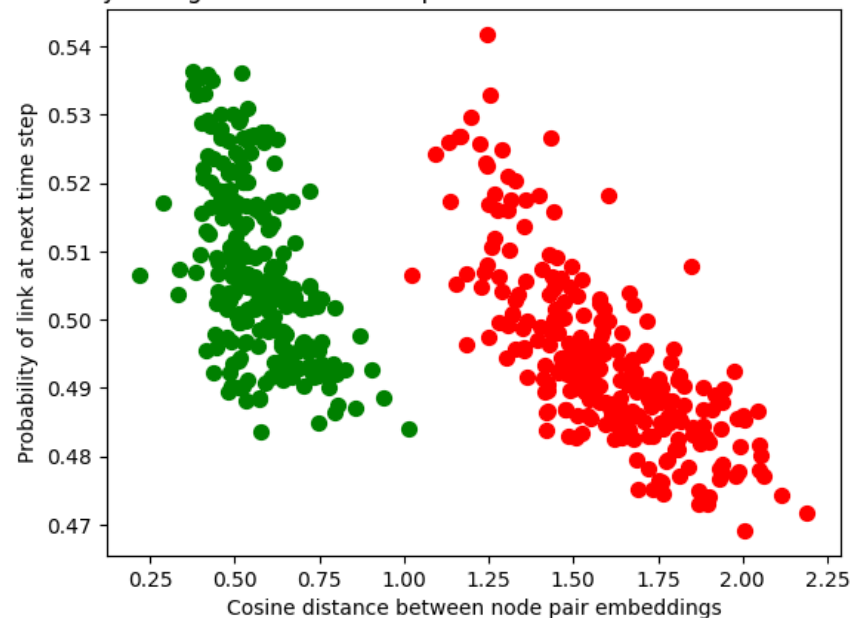
# NODE ATTRIBUTE VARIATION AND LATENT REPRESENTATIONS



The mechanism that underlies link prediction in this case is heavily influenced by node proximity in the embeddings



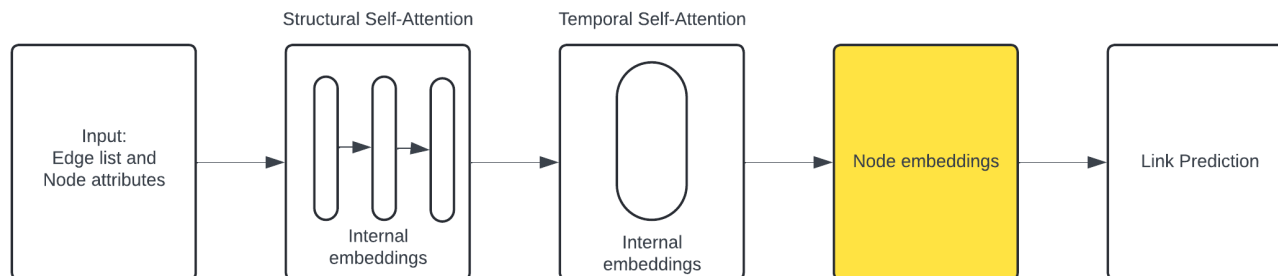
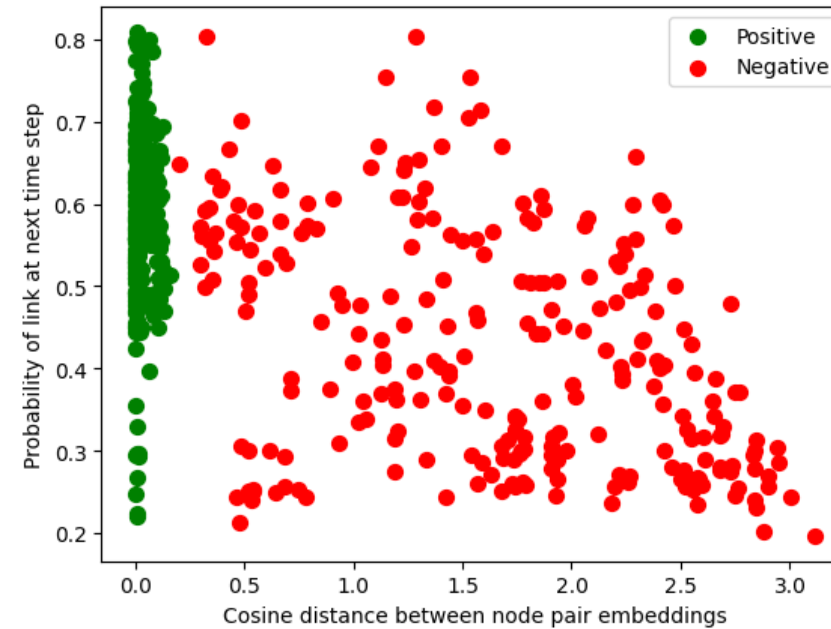
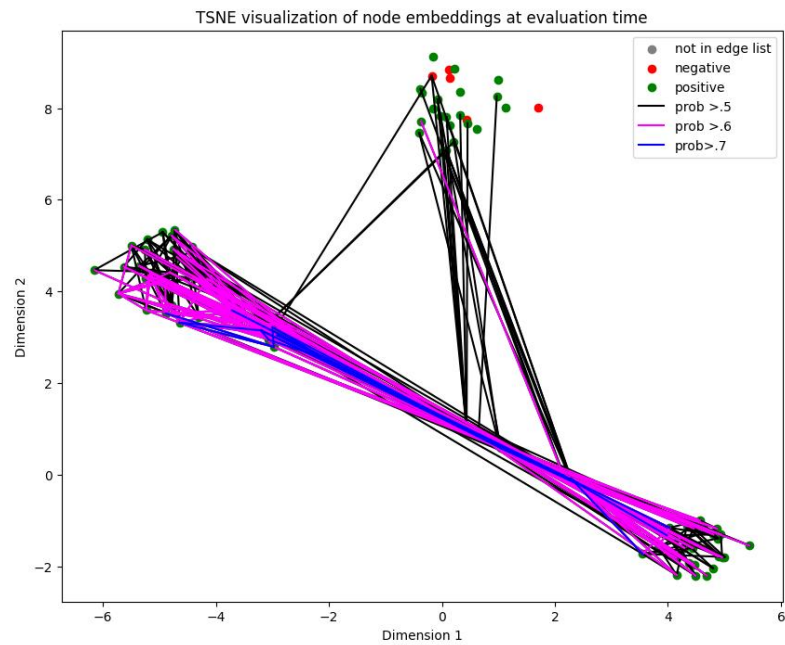
Predicted probability of edge at next time step as a function of cosine distance between node pairs



# LATENT REPRESENTATIONS AND MODEL PREDICTIONS



Probability thresholds illustrate the complexity of this relationship

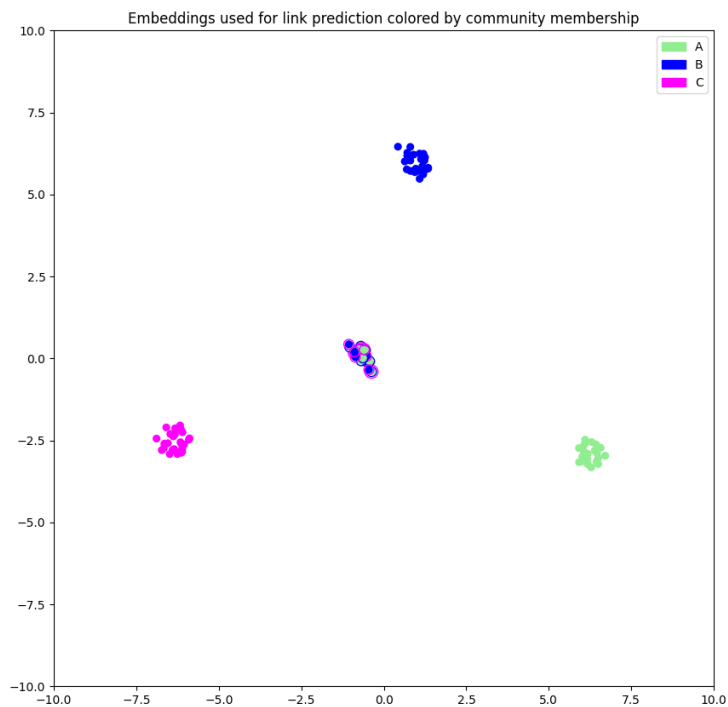




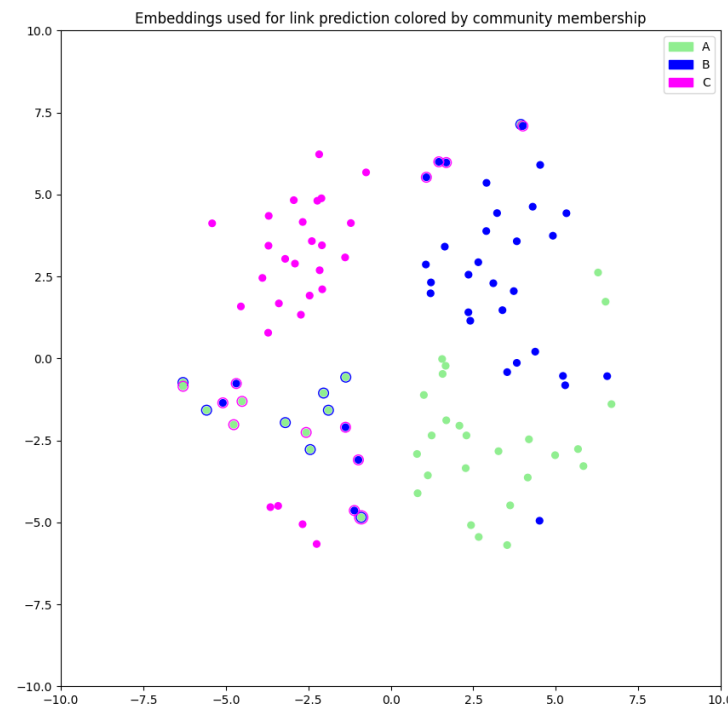
# LESSONS LEARNED

- Addition of meaningful features increases separability in latent representations
- More features isn't always better
- Even in simple models it is difficult to communicate the mechanism of link prediction in terms of model inputs

Node community membership features, trained for **100 epochs**



Node community membership features, trained for **300 epochs**





# Embeddings as a Lens for Causal Analysis

# TWO PROPOSED METHODS FOR CAUSAL EXPLANATIONS FROM GRAPH EMBEDDINGS



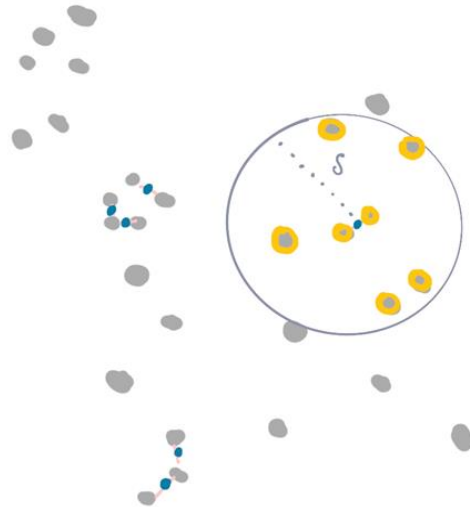
## Define causal neighborhoods based on proximity in embedding space

$$D_{ij} = \|\mathbf{e}_i - \mathbf{e}_j\|_2$$

$$\text{Link}_{ij} = \begin{cases} 1 & \text{if } D_{ij} < \tau \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{m}_{ij} = \frac{\mathbf{e}_i + \mathbf{e}_j}{2}$$

$$\mathcal{N}_{ij} = \{k \mid \|\mathbf{e}_k - \mathbf{m}_{ij}\|_2 < \delta\}$$



- Goal: Provide analyst with a mechanistic metric that can be used to inform the context of predictions in terms of familiar model inputs



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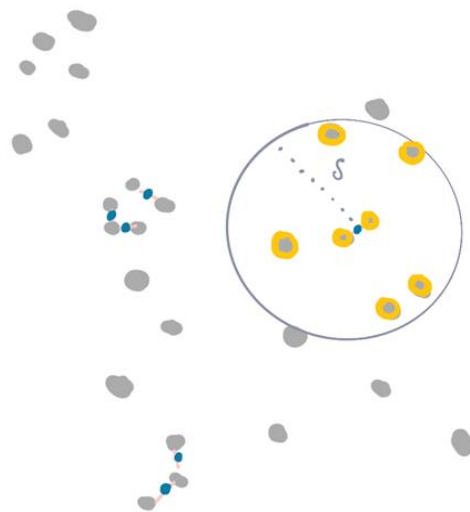
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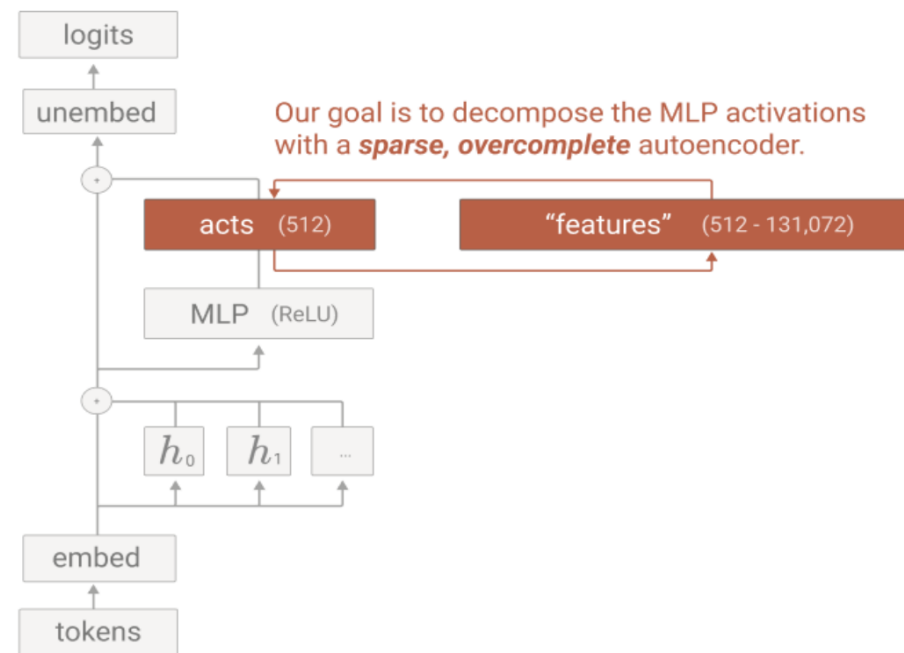
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## Generate causal diagrams via utilizing sparse autoencoder to highlight key model features influencing predictions

- Utility demonstrated by recent advances in language modeling



<https://transformer-circuits.pub/2023/monosemantic-features/index.html>

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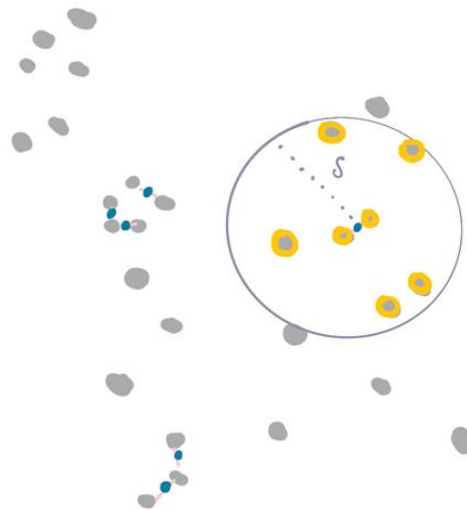
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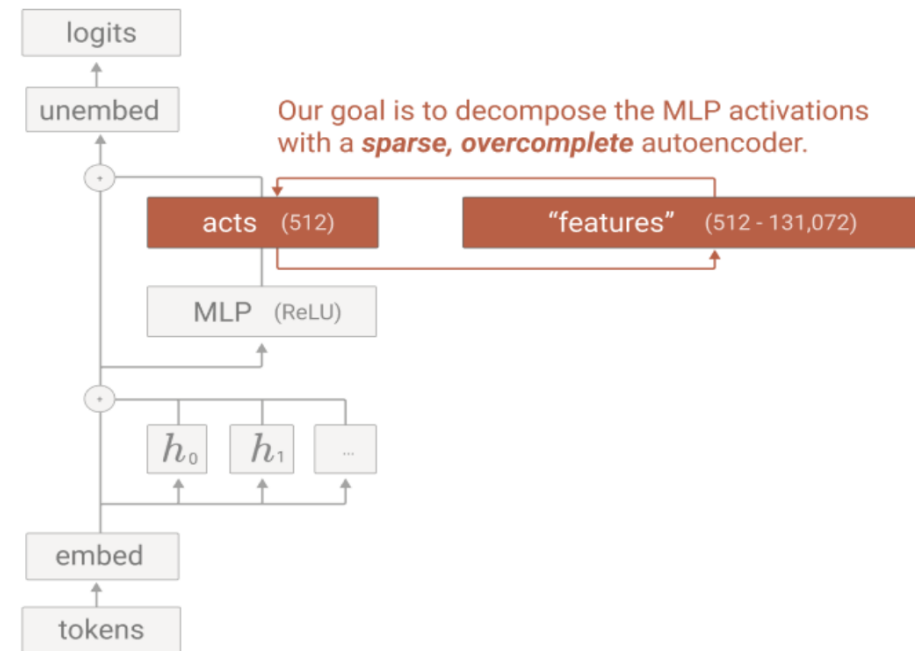
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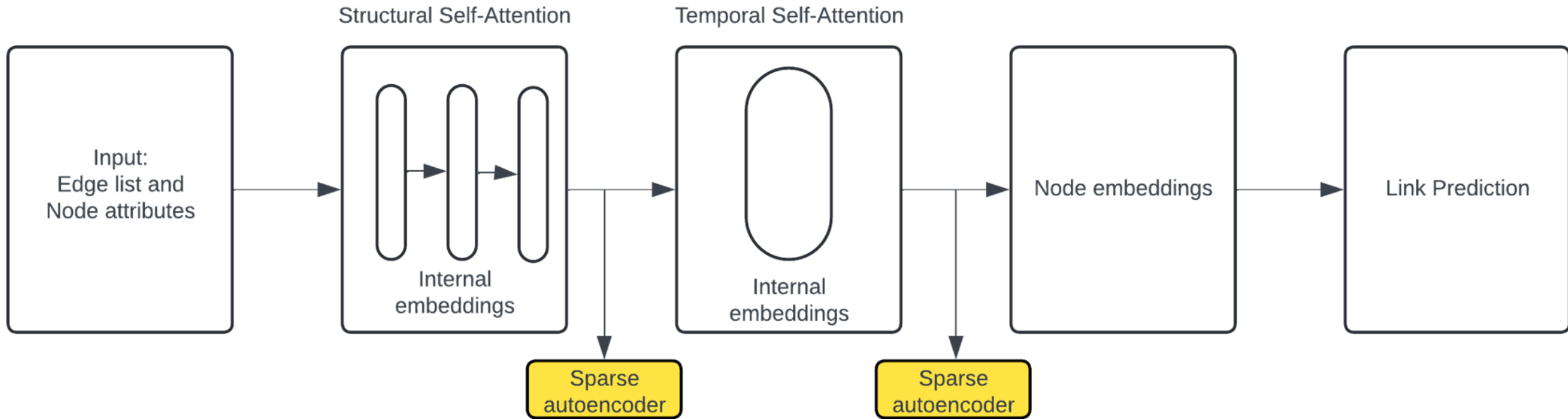
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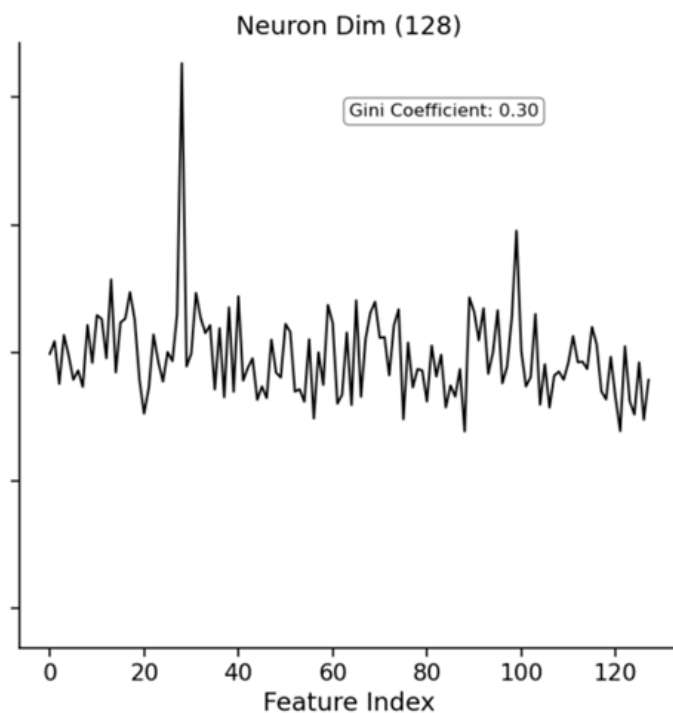
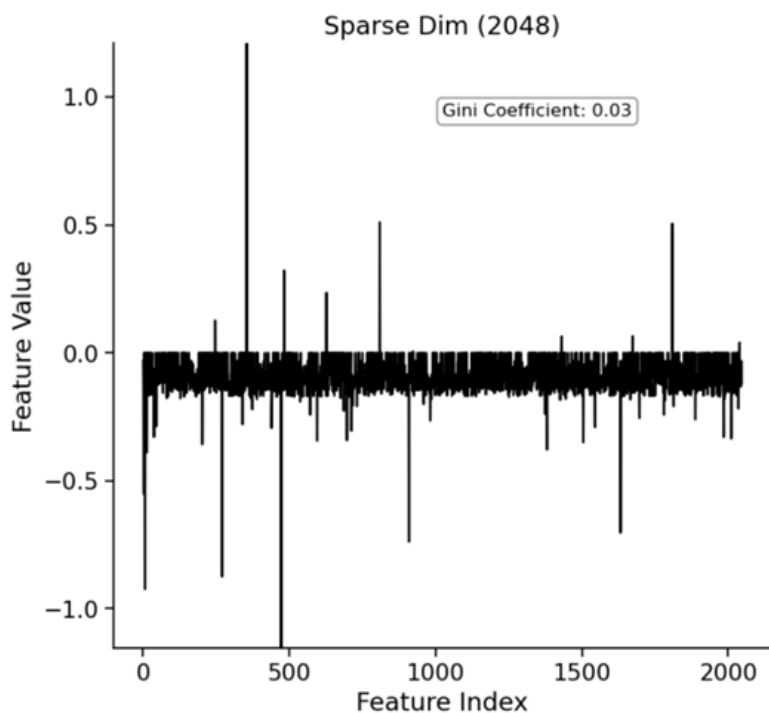
# SPARSE AUTOENCODER APPROACH



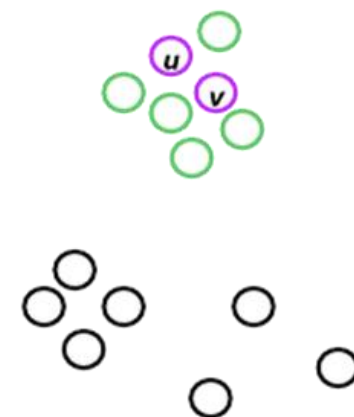


# LINK ATTRIBUTION

- Sparse and neuron dimensions provide different information
- Aim to generate more sparse representations to reduce the number of neurons responsible for placement in a given cluster



Early career biologists  
Joint appointment with national labs  
Fewer than 50 citations  
Collaborate with computer scientists



Marks, Samuel, et al. "Sparse feature circuits: Discovering and editing interpretable causal graphs in language models." *arXiv preprint arXiv:2403.19647* (2024)

# SUMMARY

- Graph representations are ubiquitous in modeling complex systems but lack causal explanation
- Graph attention networks achieve state-of-the-art performance while providing latent representations that can be leveraged to extract mechanistic interpretability
- Promising recent results in large language modeling provide a good starting point for generating causal explanations
- Causal explanations are only as good as the representations they came from





Questions?