Nonparametric Sparse Learning of Nonlinear Dynamical Systems

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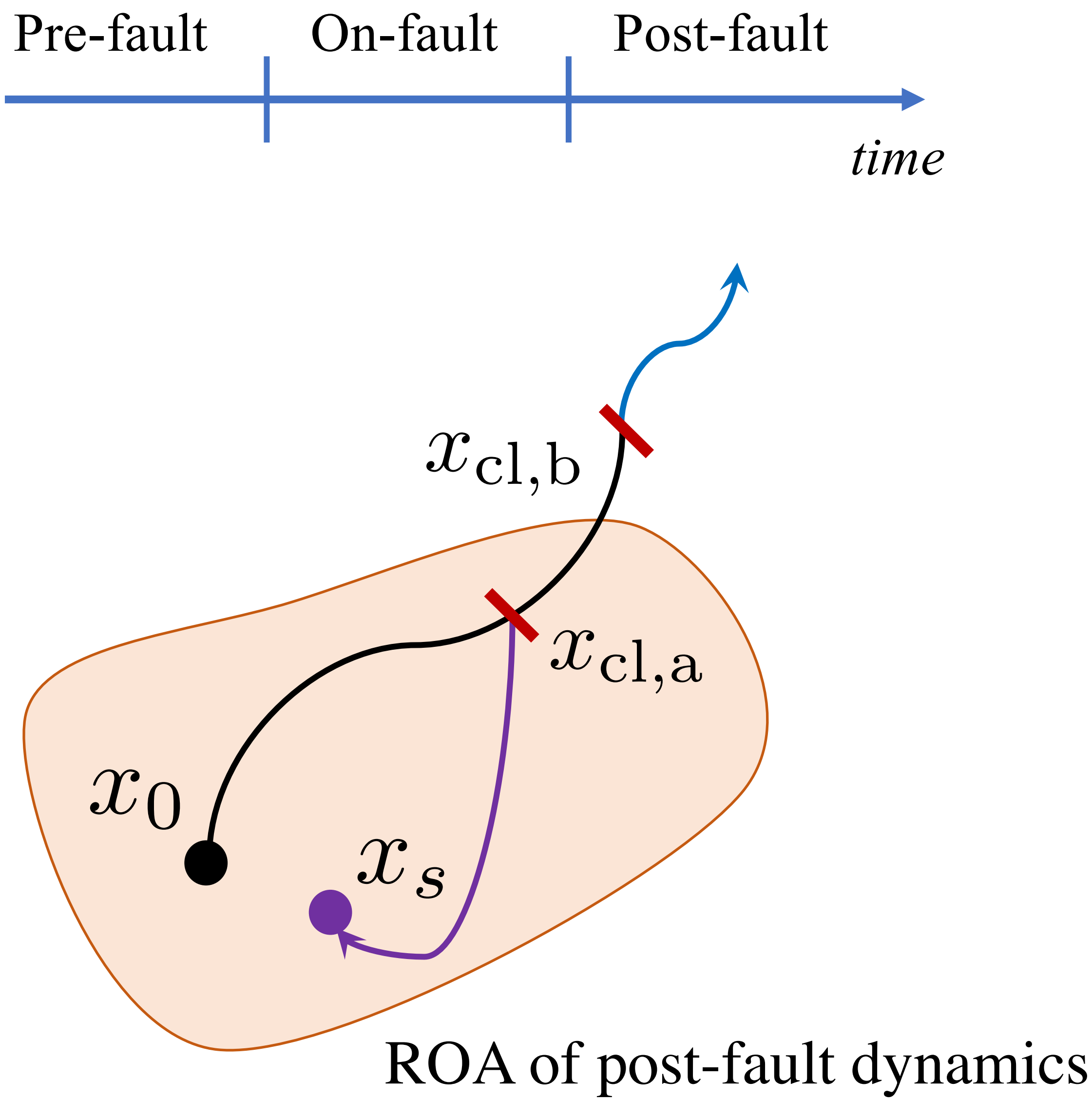
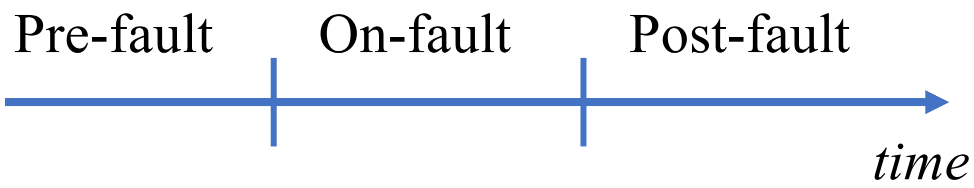
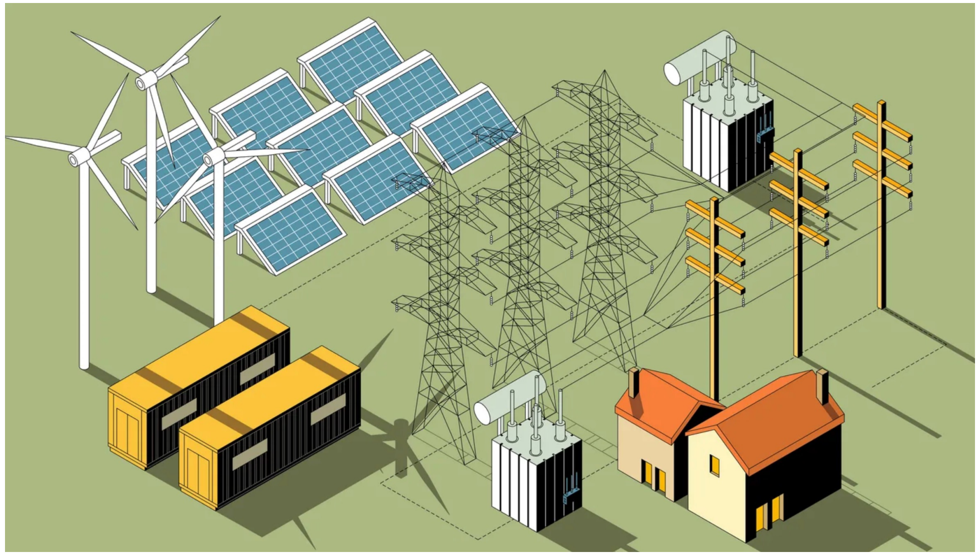
Data Driven Transient Stability Analysis



Data Driven Transient Stability Analysis via Koopman Operator



- Numerical integration / energy functions
- From data?



Markovian dynamical system

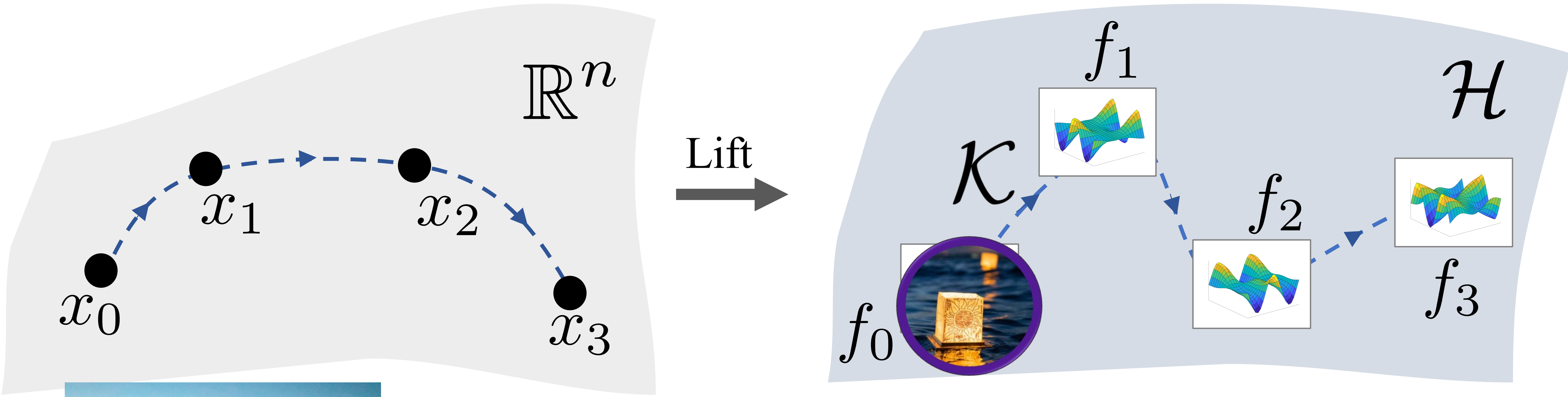
$$\mathbb{P}\{x_{t+1} \in \mathbb{A} | x_t = x\} = \int_{\mathbb{A}} p(y|x) \mathrm{d}y,$$

Koopman operator

$$(\mathcal{K} f)(x) = \int p(y|x) f(y) \mathrm{d}y.$$

$\mathcal{K}$ as a representation of system dynamics

Koopman Operator $(\mathcal{K}f)(x) = \mathbb{E}[f(X_{t+1})|X_t = x]$ [Koopman 31]

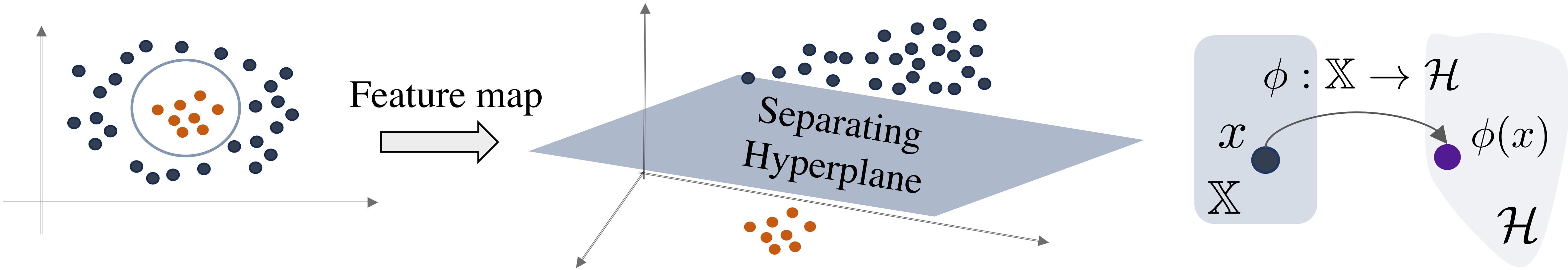


Given $\{x_0, \ldots, x_m\}$, learn $\widehat{K}$ s.t.

$$\|\widehat{K} - \mathcal{K}\|_{\mathrm{op}} \leq \varepsilon \quad \text{w.h.p.}$$

in the reproducing kernel Hilbert space

Koopman Operators on Reproducing Kernel Hilbert Space

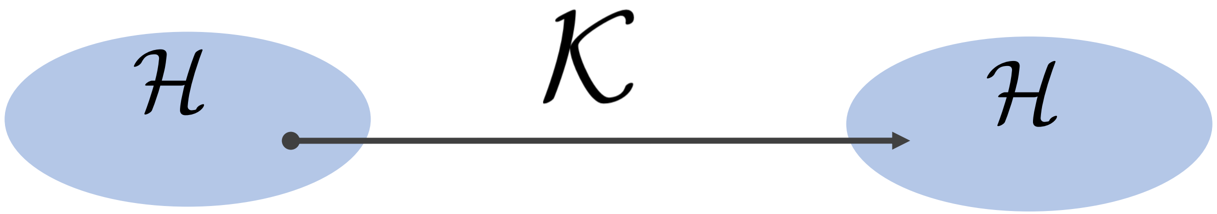


Consider a Reproducing Kernel Hilbert Space (RKHS) $\mathcal{H}$ with kernel $\kappa(\cdot,\cdot)$

Canonical feature map: $\phi(x) = \kappa(\cdot, x)$

Denote the covariance operators as $C_{XX^+} := \mathbb{E}[\phi(X) \otimes \phi(X^+)]$, $C_{XX} := \mathbb{E}[\phi(X) \otimes \phi(X)]$

When interacting with RKHS, the Koopman operator can be written as $\mathcal{K} = C_{XX}^{\dagger} C_{XX^+}$ [Klus ’20]



Nonparametric Sparse Learning of Nonlinear Dynamical Systems

The regularized Koopman operator $\mathcal{K}_\lambda = (C_{XX} + \lambda I)^{-1} C_{XX^+}$

Given a dataset $D = \{(x_1, x_1^+), \ldots, (x_M, x_M^+)\}$

Model complexity is captured by $|D| = M$

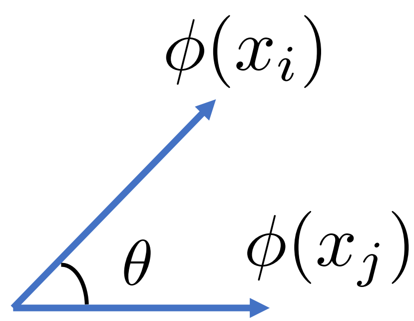
Empirical estimates of covariance operators

$$C_{X^+X}^M = \frac{1}{M} \sum_{i=1}^{M} \phi(x_i^+) \otimes \phi(x_i), \quad C_{XX}^M = \frac{1}{M} \sum_{i=1}^{M} \phi(x_i) \otimes \phi(x_i).$$

The regularized empirical estimate are $K_\lambda^M = (C_{XX}^M + \lambda I)^{-1} C_{XX^+}^M$

Prune D to construct a sparse dictionary $D_\gamma \subseteq D$ via *the coherence condition*

$$\frac{|\langle \phi(x_i), \phi(x_j) \rangle_{\mathcal{H}}|}{\|\phi(x_i)\|_{\mathcal{H}} \|\phi(x_j)\|_{\mathcal{H}}} \times \frac{|\langle \phi(x_i^+), \phi(x_j^+) \rangle_{\mathcal{H}}|}{\|\phi(x_i^+)\|_{\mathcal{H}} \|\phi(x_j^+)\|_{\mathcal{H}}} \leq \gamma, \forall (x_i, x_i^+), (x_j, x_j^+) \in D_\gamma$$



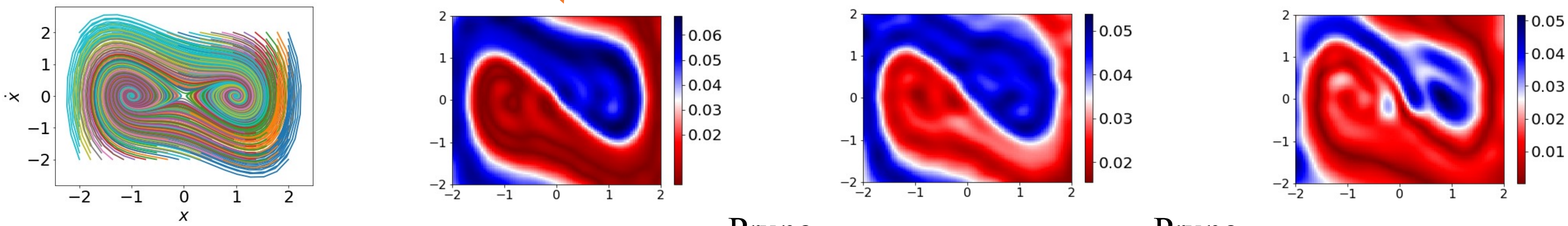
Sparse kernel Koopman operator $\widehat{K}_\lambda = (\widehat{C}_{XX} + \lambda I)^{-1} \widehat{C}_{XX^+}$

Model complexity is captured by $|D_\gamma| \ll |D|$

Nonparametric Sparse Learning of Nonlinear Dynamical Systems

Sparse Koopman operator $\widehat{K}_\lambda = (\widehat{C}_{XX} + \lambda I)^{-1}\widehat{C}_{XX^+}$

Leading eigenfunction



Duffing oscillator $|D| = 1354$ Prune → $|D_\gamma| = 760$ Prune → $|D_\gamma| = 274$

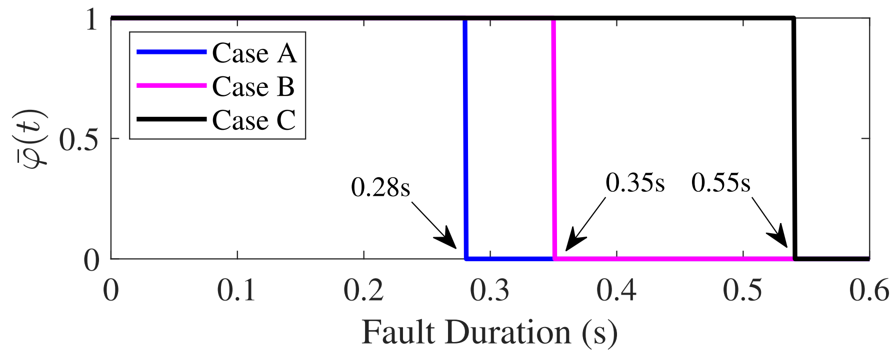
Figure: Estimated regions of attraction with sparse dataset $D_\gamma \subseteq D$. [Hou et al. 21]

An example of theoretical guarantees: high probability bound with IID samples. [Hou et al ICML 23]

$$\|\mathcal{K}_\lambda - \widehat{K}_\lambda\|_{\text{op}} \leq [\text{sampling error}(\frac{1}{\sqrt{M}}) + \text{sparsification error}(\sqrt{1-\gamma^2})]\mathcal{O}(\frac{1}{\lambda^2})$$

- IID samples → Trajectory-based sampling. [Hou et al ICML 23]
- Batch learning → Learning from steaming samples with last-iterate convergence guarantees. [Hou et al 24b]
- A similar framework for learning generators of diffusion processes. [Hou et al ICML 23]

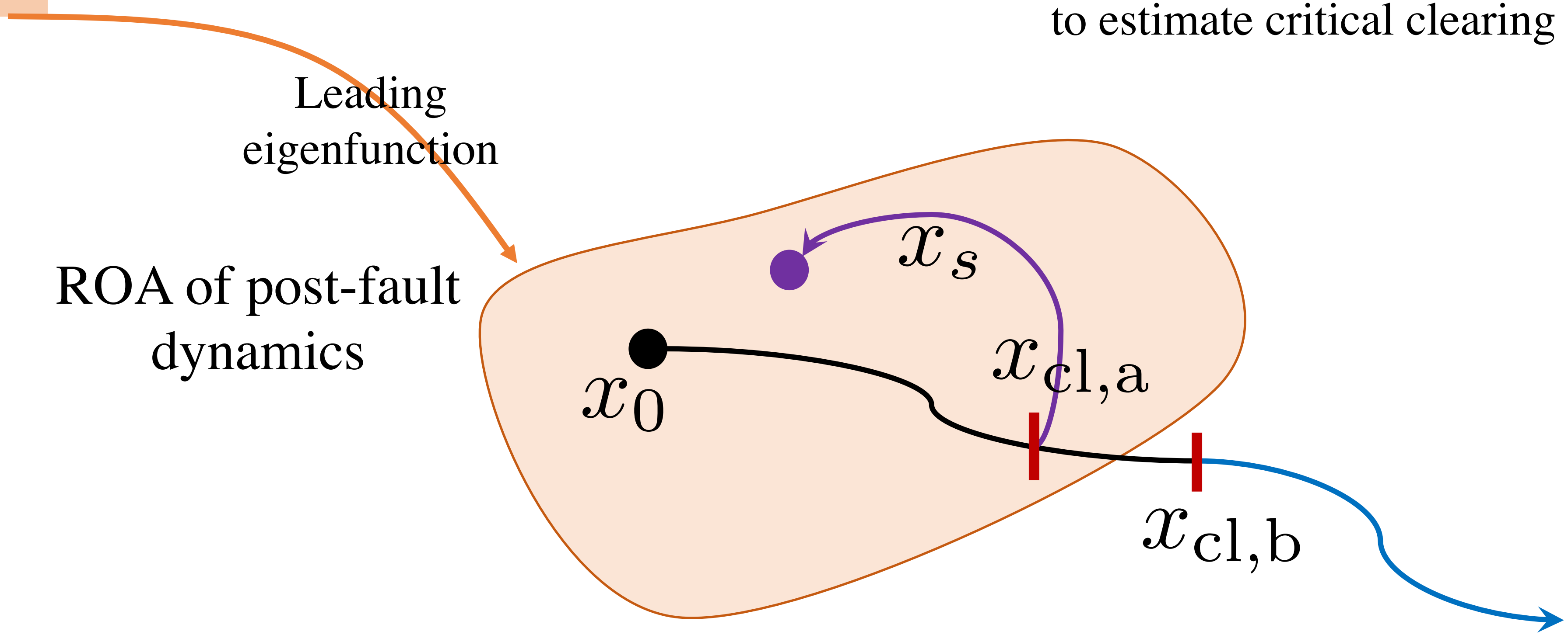
Koopman Operator for Transient Stability Analysis of Power Systems



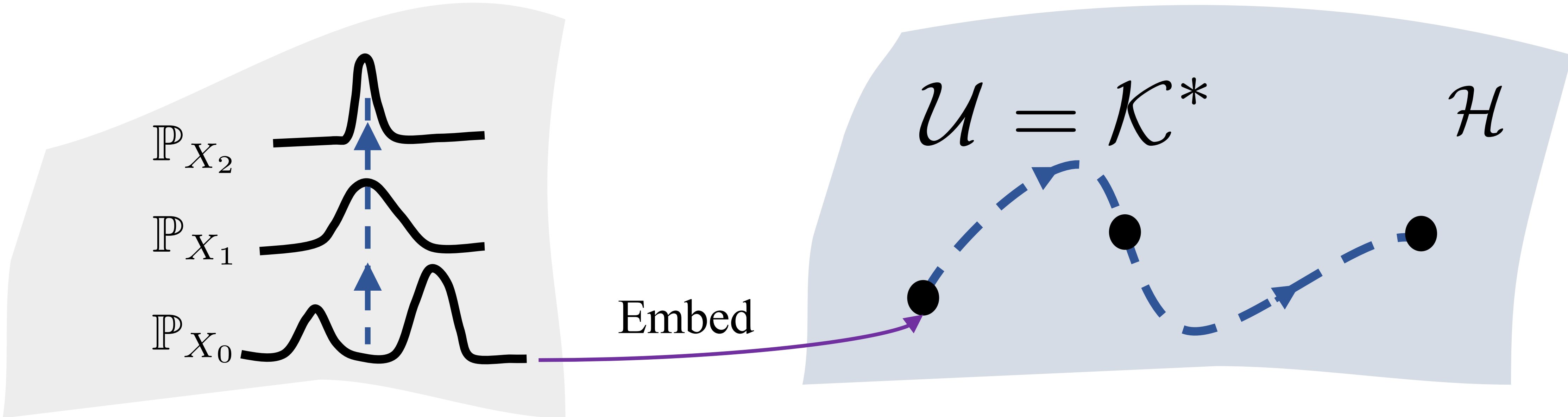
Sparse kernel Koopman operator

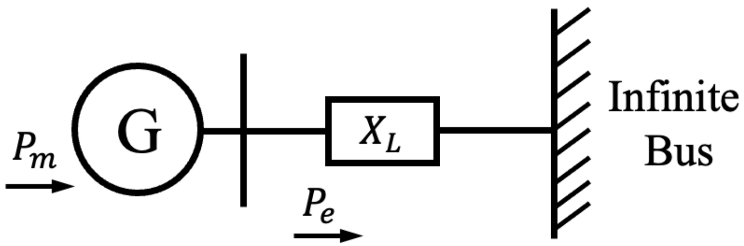
$$\widehat{K}_\lambda = (\widehat{C}_{XX} + \lambda I)^{-1}\widehat{C}_{XX^+}$$

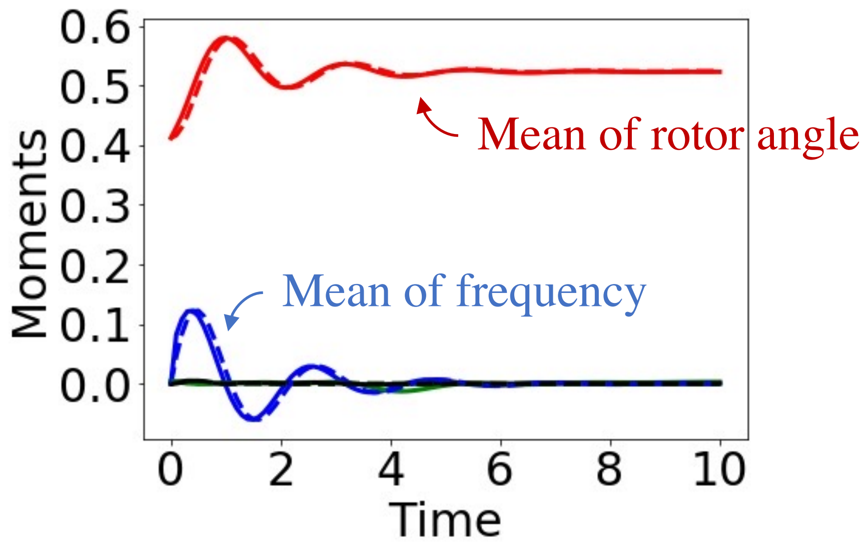
IEEE 39 bus system: Evaluate the leading eigenfunction of K along on-fault trajectories to estimate critical clearing time. [Hou et al. 24a]



Propagating Uncertainty in Initial Conditions [Hou et al. Physica D 24]







Single Machine Infinite Bus Systems:

uncertainties in the initial conditions are rotor angles of the generator

Summary of Contributions

Non-parametric identification of nonlinear systems with controlled growth in model complexity and finite-sample performance guarantees.

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[Hou et al. 24b] Boya Hou, Sina Sanjari, Alec Koppel, and Subhonmesh Bose. Compressed online learning of conditional mean embedding. arXiv preprint arXiv:2405.07432, 2024

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Thank you !

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