Domain decomposition-based coupling of subdomain-local reduced order models (ROMs) in fluid mechanics using the Schwarz alternating method

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ABSTRACT

This talk will describe a novel approach for creating adaptive hybrid models through domain decomposition (DD) and the Schwarz alternating method (SAM) [1], towards enabling predictive digital twins. In this approach, the solution on the full domain is obtained via an iterative process in which a sequence of subdomain-local problems are solved, with information propagating between subdomains through transmission boundary conditions (BCs). The models being coupled can be subdomain-local full order models (FOMs) and/or subdomain-local reduced order models (ROMs). We will present some new developments involving SAM for the specific case of cell-centered finite volume (CCFV) discretizations applied to fluid problems [2]. We demonstrate that, for CCFV Methods and a non-overlapping DD, it is possible to obtain a stable and accurate coupled model utilizing Dirichlet-Dirichlet (rather than Robin-Robin or alternating Dirichlet-Neumann) transmission BCs on the subdomain boundaries. We additionally explore the impact of boundary sampling when utilizing the Schwarz alternating method to couple subdomain-local hyper-reduced Proper Orthogonal Decomposition (POD)/Least-Squares Petrov-Galerkin (LSPG) ROMs. Our numerical results on three fluid conservation law problems (2D Burgers equation, 2D shallow wáter equations, 2D compressible Euler equations) suggest that the proposed methodology has the potential to improve ROM accuracy by enabling the spatial localization of these models via domain decomposition, and achieve up to two orders of magnitude speedup over equivalent coupled full order model solutions and moderate speedups over analogous monolithic solutions. Time permitting, I will discuss some new developments in extending SAM to enable the coupling of subdomain-local non-intrusive ROMs constructed via Operator Inference (OpInf) [3], and present some perspectives towards enabling on-the-fly switching between subdomain-local models of varying fidelities.

REFERENCES

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