

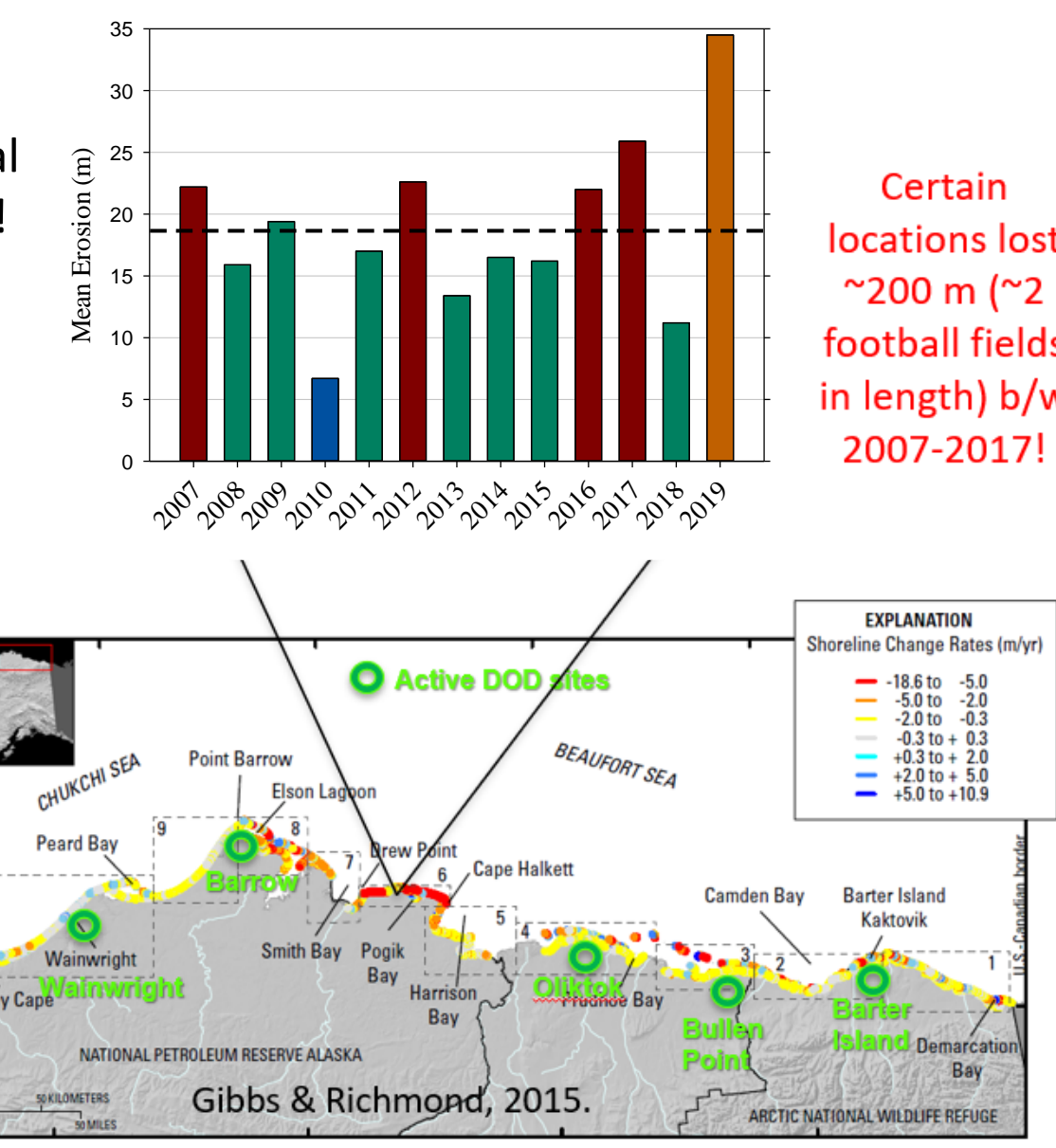
## Motivation

The Arctic is warming at **4 times** the rate of the global average resulting in **accelerated rates of coastal erosion!**

- Primary culprit is **loss of Arctic sea ice**: since 1979 sea ice has lost 51% in area and 75% in volume
  - Increasing **ice-free season**
  - Increasing **wave energy** and **storm surge**
  - Increasing **sea water temperatures**

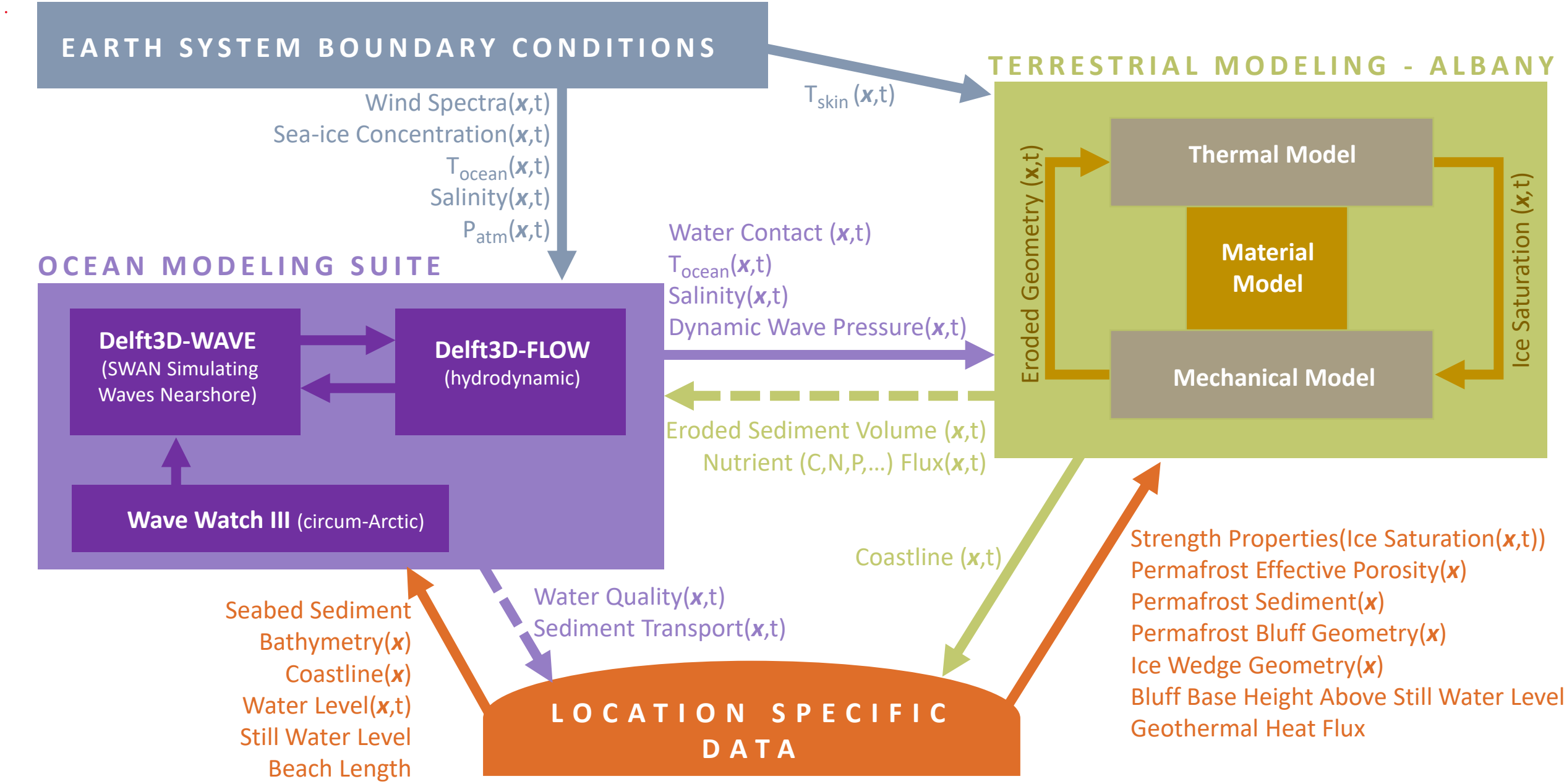
### Erosion is threatening:

- Coastal communities**: threatened with displacement
- Coastal infrastructure**: active DoD sites, including toxic waste sites, in northern Alaska
- Global carbon balance**: permafrost stores greenhouse gases (CO<sub>2</sub>, CH<sub>4</sub>, NO<sub>2</sub>).



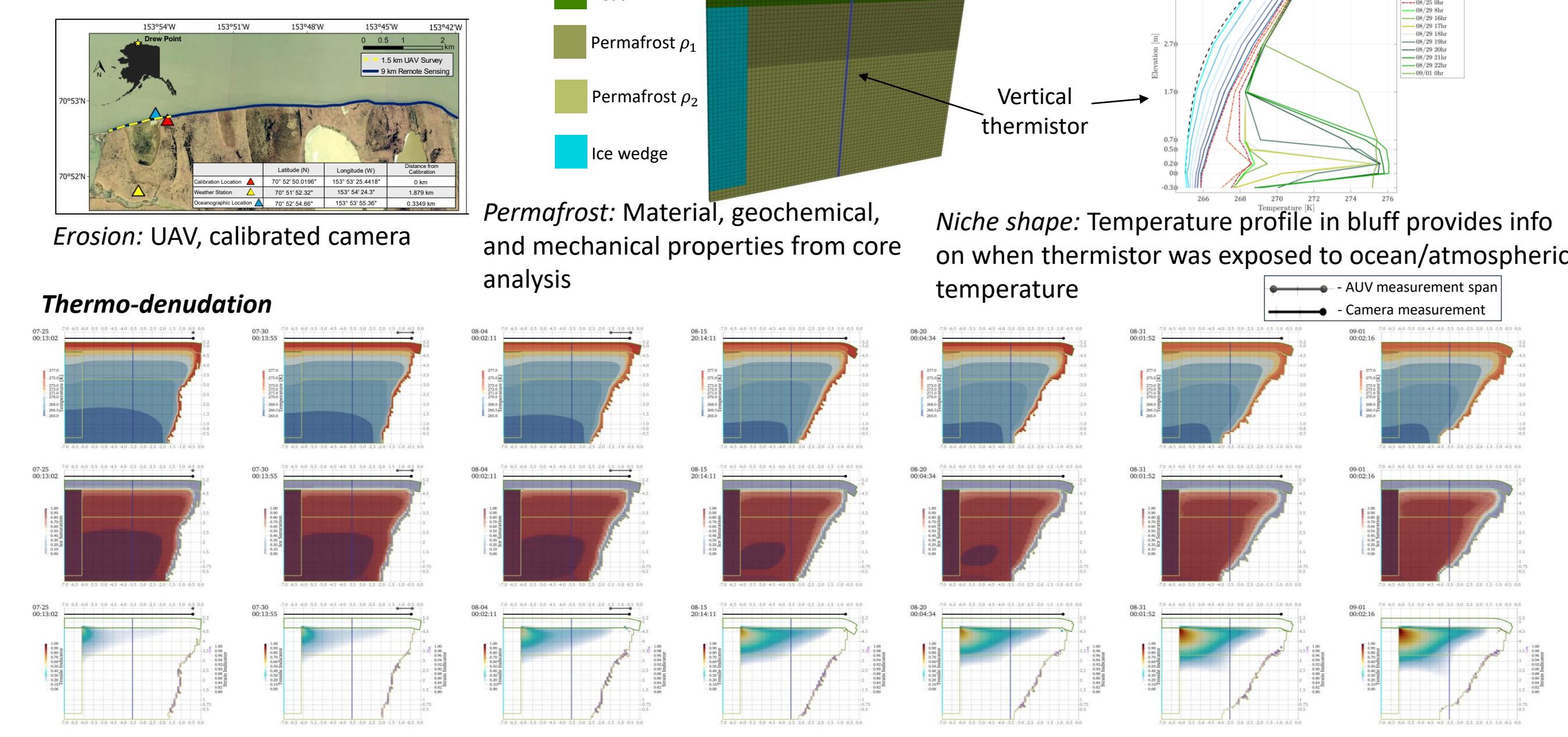
## ACE Model: Component Coupling

ACE Model development supported by the Laboratory Directed Research and Development program at Sandia National Laboratories.



## Calibration Case & Results

Summer 2018 at Drew Point AK



## Terrestrial Model

### Thermal Model

\*Terrestrial model components and coupling implemented in Albany

### Coupling (Schwarz framework)

### Mechanical Model

Thermal PDE's evolve temperature and ice saturation in permafrost

→ PDE's respond to BC's as well as a salinity enhanced melting

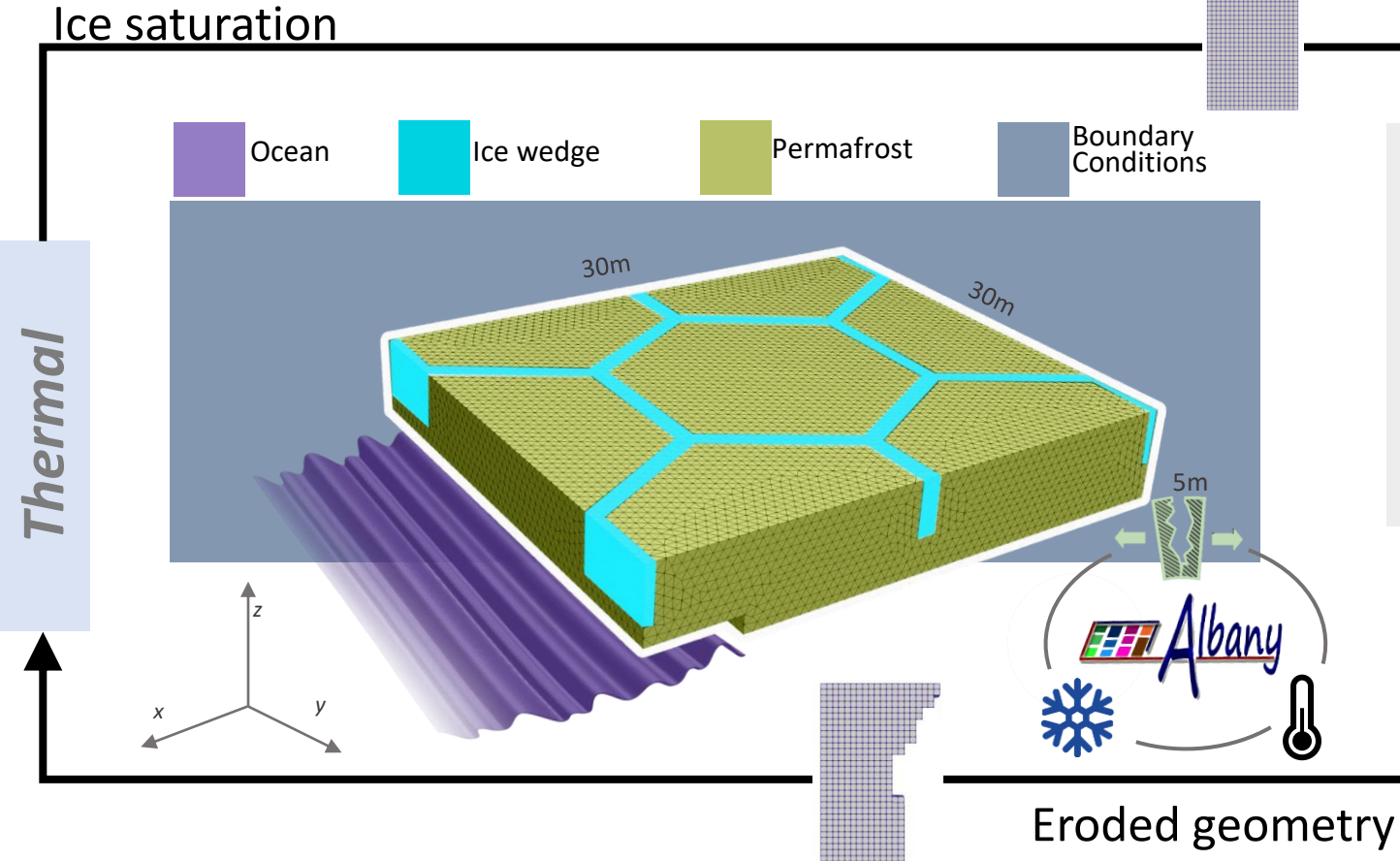
→ As material fails, new portions of permafrost are exposed to B.C.s

• **Transient heat conduction** in a non-homogeneous porous media with water-ice phase change:

$$(\rho c_p + \bar{\theta}) \frac{\partial T}{\partial t} = \nabla \cdot (K \cdot \nabla T)$$

where  $\bar{\theta} := \rho_f L_f \frac{\partial f}{\partial T}$  incorporates phase changes through soil freezing curve,  $\frac{\partial f}{\partial T}$ .

$\bar{\rho}$ : density from mixture model  
 $\bar{c}_p$ : specific heat from mixture model  
 $K$ : thermal diffusivity tensor  
 $\rho_f$ : ice density  
 $L_f$ : latent heat of water-ice phase change  
 $f$ : ice saturation ( $\in [0,1]$ )  
 $\frac{\partial f}{\partial T}$ : soil freezing curve (depends on salinity, sediment fraction)



Mechanical PDE's based on finite deformation plasticity model

- 3x3 tensor of compressive, tensile, and shear components computed everywhere in the model (J2 class)

→ Domain will deform according to computed stress

→ **Dynamic pressure from ocean waves** computed as an additional stress on boundary cells

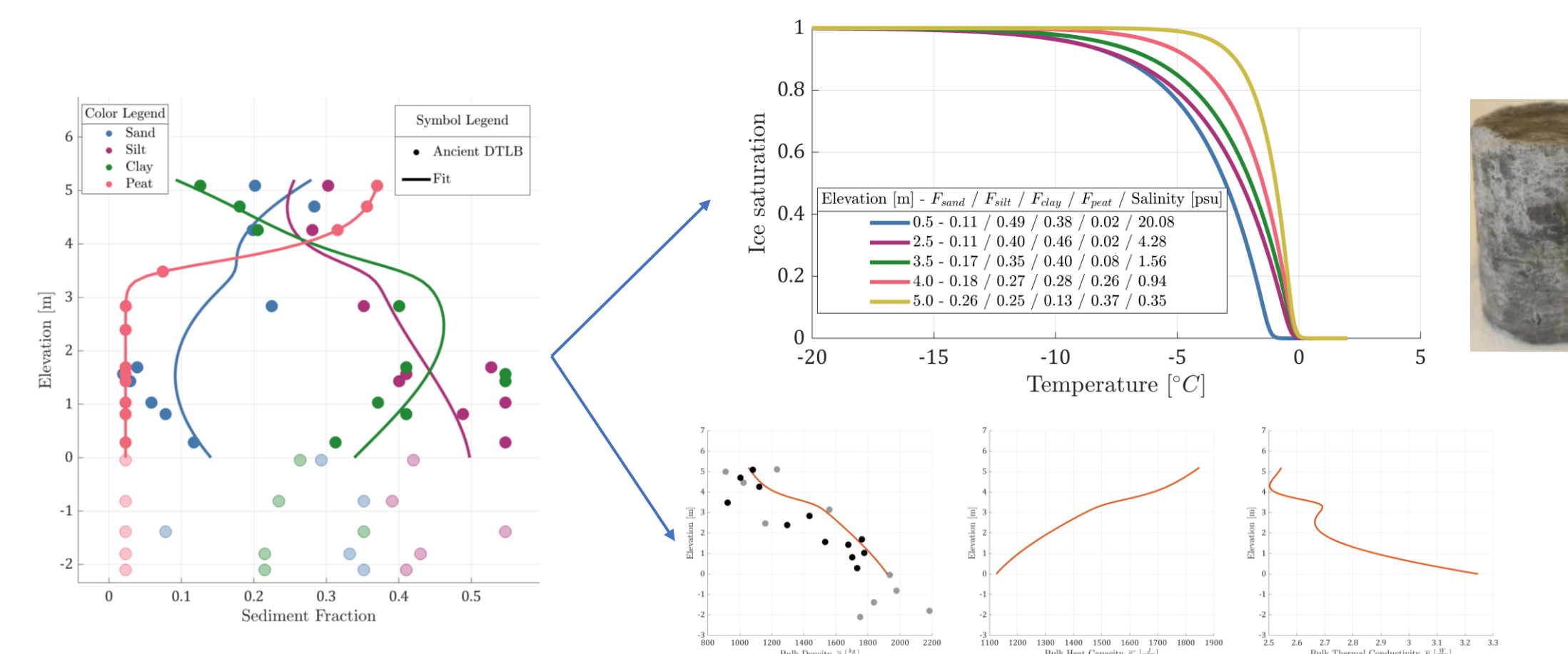
Finite deformation **time-dependent** variational formulation for **solid mechanics problem** obtained by minimizing the energy functional:

$$\Phi[\varphi] := \int_{\Omega} A(F, Z) dV - \int_{\Omega} \rho B \cdot \varphi dV - \int_{\partial\gamma_{\Omega}} T \cdot \varphi dS$$

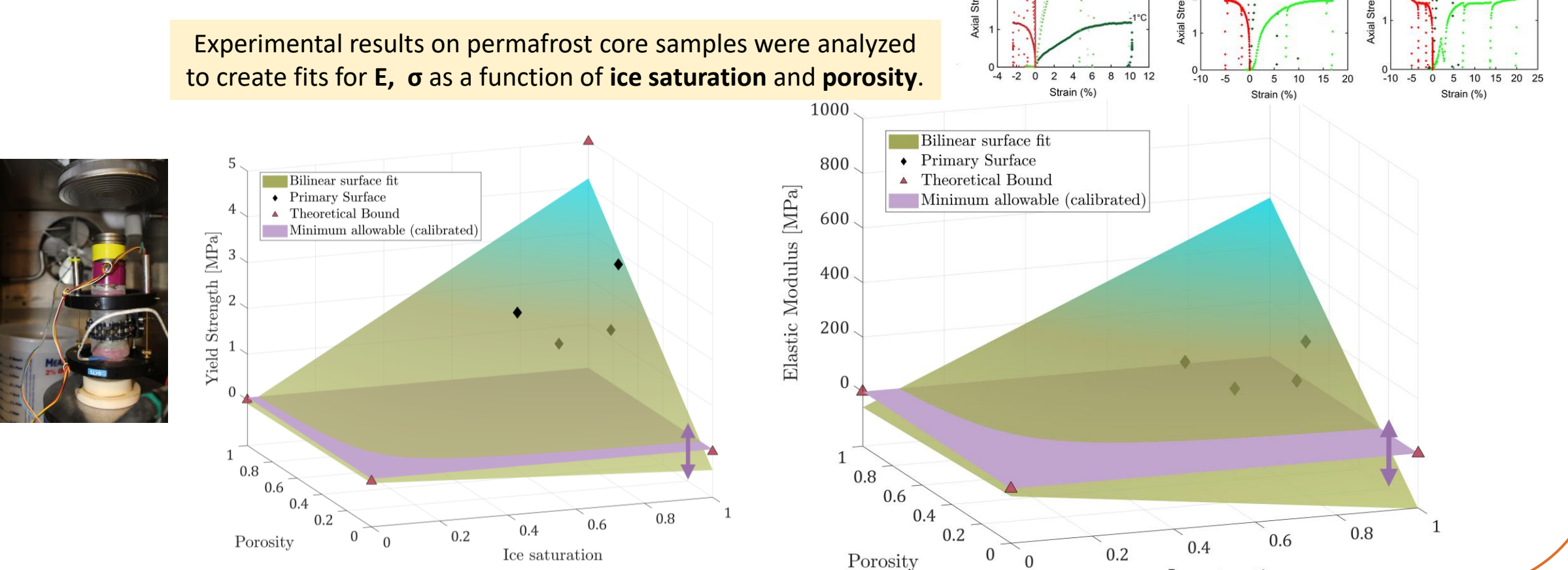
$A(F, Z)$ : Helmholtz free-energy density  
 $Z$ : material variables  
 $F$ : deformation gradient ( $\nabla\varphi$ )  
 $\rho$ : density  
 $B$ : body force  
 $T$ : prescribed traction

## Material Models

- Developed **thermal** properties (conductivity, heat capacity, etc.) from mixture models of constituent material properties
- Soil freezing curve dependent on sediment fraction, salinity



- Constitutive relationships for the **mechanical** model
- Require stress-strain curves up to failure as function of ice saturation and porosity for local permafrost samples
- Asymmetric yield stress under tension and compression



## Erosion

Domain changes geometry (through mesh adaptation) according to following **erosion/failure criteria**:

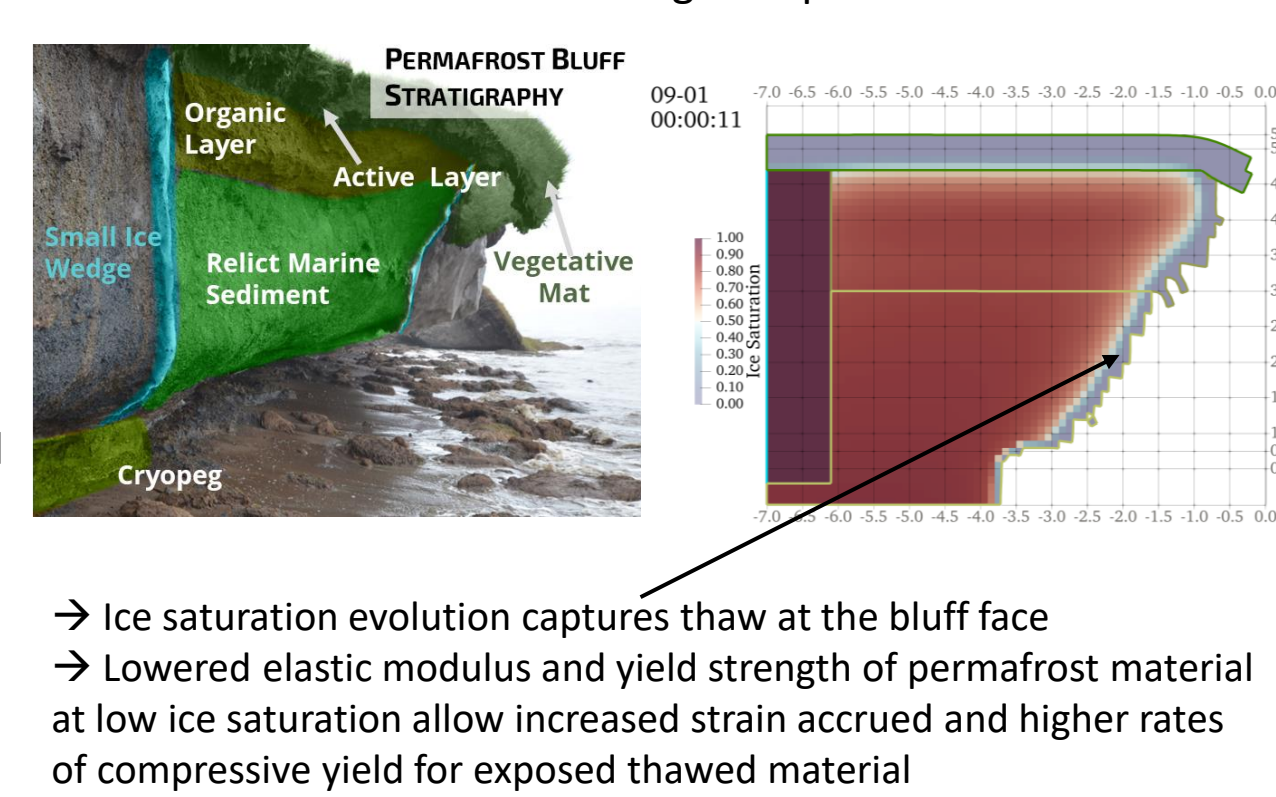
**Stress criterion**: remove element when critical value of yield stress in tension or compression is surpassed (typically high ice content)

**Strain criterion**: remove element when it has deformed beyond a critical value; defined as a function of peat content (typically most active in regions of low ice saturation)

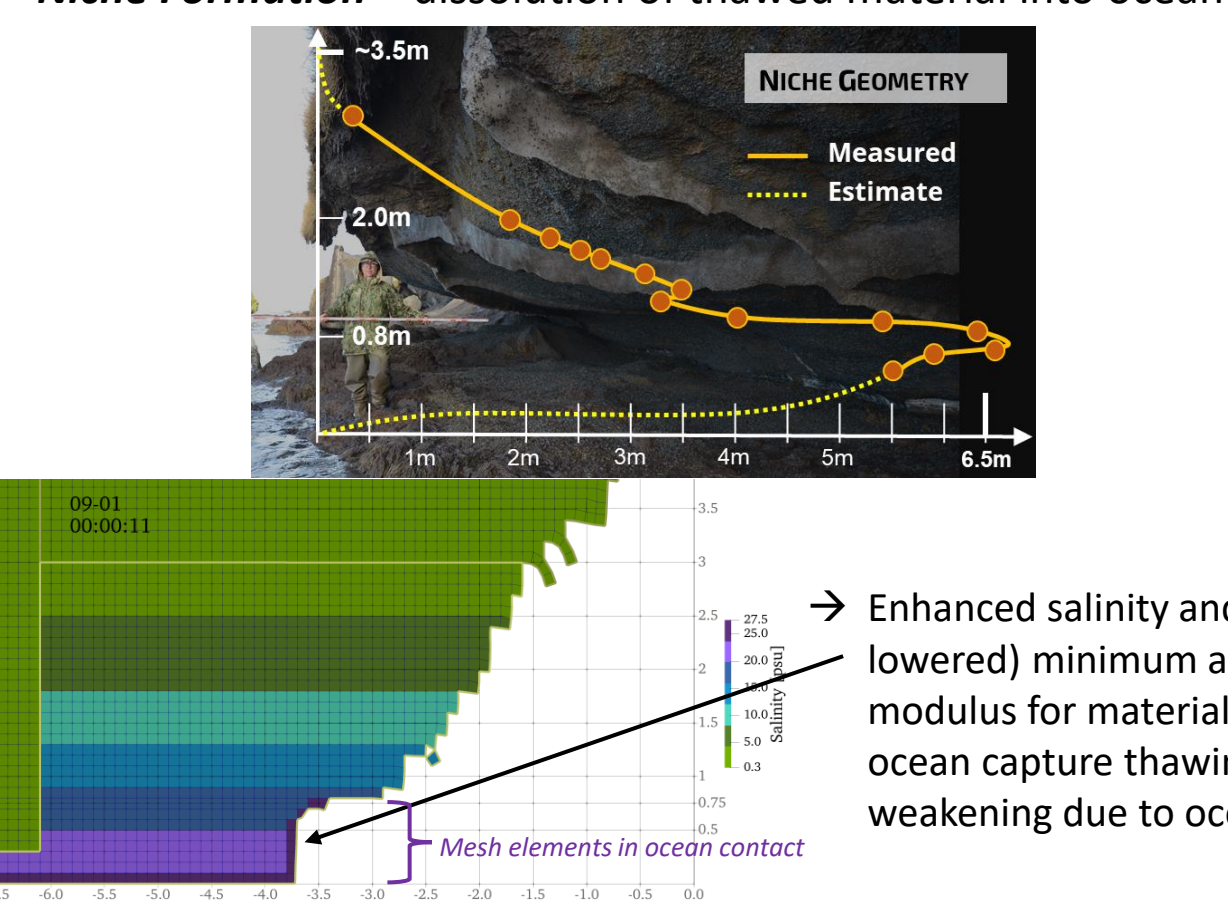
**Kinematic criterion** (solver stability): remove element when material has tilted/displaced excessively

### Distinct mechanisms of erosion:

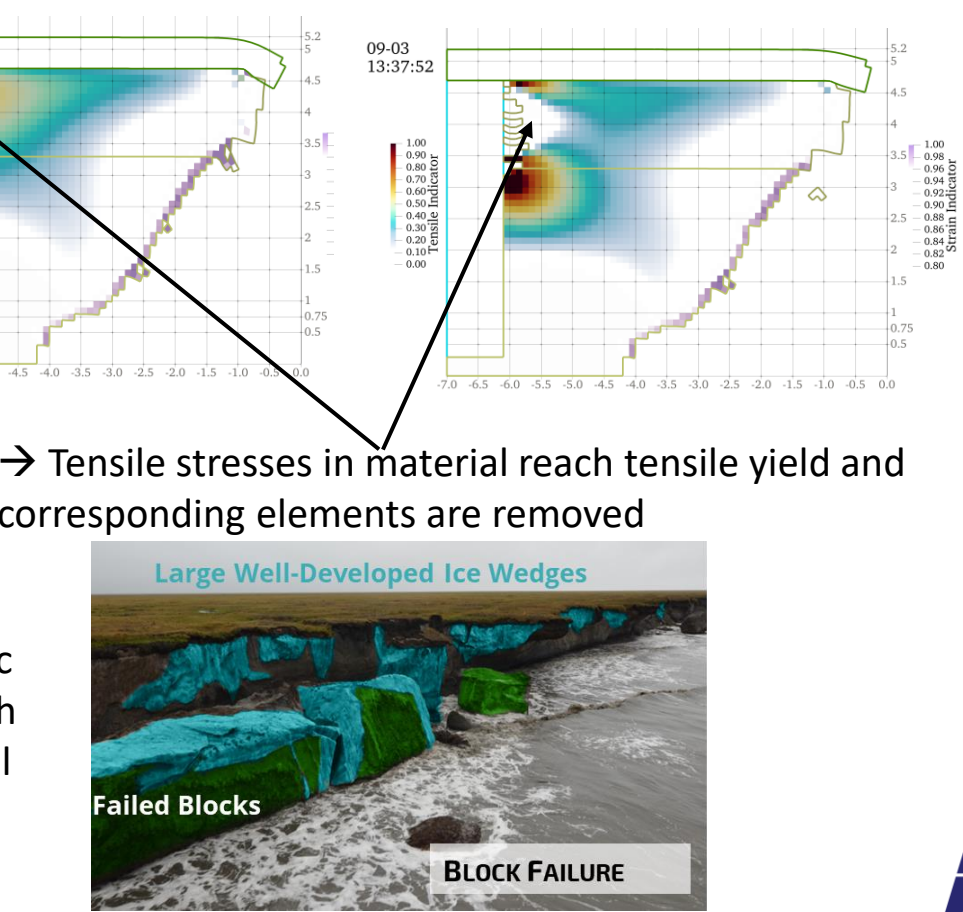
**Thermo-denudation** – thawing of exposed material



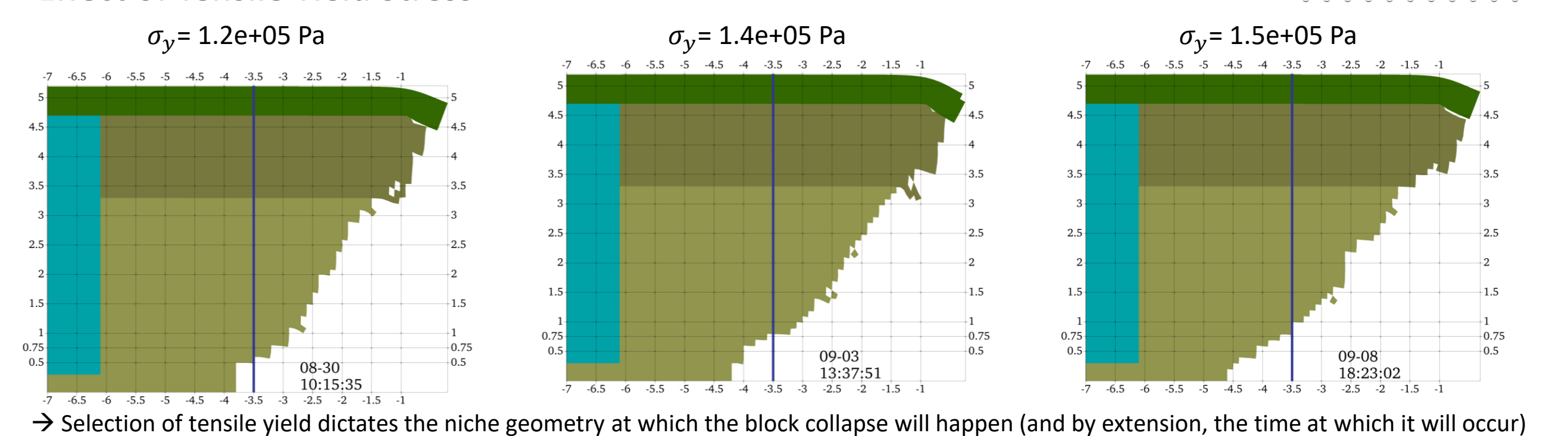
**Niche Formation** – dissolution of thawed material into ocean



**Block Failure** – tensile failure of frozen material



### Effect of Tensile Yield Stress



→ Selection of tensile yield dictates the niche geometry at which the block collapse will happen (and by extension, the time at which it will occur)

**For further information:**  
 Frederick, J. M., A. Mota, I. Tezaur, and D. L. Bull (2021), A thermo-mechanical terrestrial model of Arctic coastal erosion, *Journal of Computational and Applied Mathematics*, Vol. 397, doi:10.1016/j.cam.2021.113533.  
 Bull et al. (2020), Arctic Coastal Erosion: Modeling and Experimentation, SAND2020-10223, Sandia National Laboratories, Albuquerque, NM.  
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